



# **Labor Markets, Economic Policy, & Business Cycles**

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# Biographical Note

Luís Alexandre Barbosa Guimarães was born on April 13, 1989, in Vila Nova de Famalicão, Portugal.

Luís concluded his undergraduate studies in Economics in 2010, at the Faculty of Economics of the University of Porto (FEP), with the final grade of 17 out of 20. In 2009, he was a summer intern at the Bank of Portugal, and was awarded an Integration to Research Grant (BII) by LIAAD financed by the Foundation for Science and Technology (FCT). In 2010, before finishing his undergraduate studies, he was a tutor for PALOP (Portuguese speaking African countries) students for Mathematics II.

After graduation, Luís entered the PhD in Economics of the Faculty of Economics of the University of Porto, with a research grant by the FCT. After completing the scholar component of the PhD with a grade of 18 out 20, he started his thesis under the supervision of Alper Çenesiz. During his PhD, he presented his work in several seminars. He also contributed to a project for the Ministry of Economy about active employment policies, and co-authored a chapter on the first volume of the *CIM-MPE* and a comment paper at the *Journal of International Money and Finance*.

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# Resumo

A minha tese é composta por três ensaios. No primeiro ensaio, eu sou o primeiro a relacionar a sorte com a proteção de emprego: eu estudo os efeitos da proteção de emprego na importância da sorte, mérito e esforço em determinar a desigualdade no rendimento. Para esse fim, eu desenvolvo um modelo com trabalhadores e empresas heterógeneas cuja interação está sujeita a fricções de *matching*. No modelo, um aumento da proteção de emprego aumenta a importância da sorte e diminui a importância do mérito em gerar desigualdade do rendimento.

No segundo ensaio, eu e o Alper Çenesiz, estudamos os efeitos de política monetária no produto agregado e produtos sectoriais. O modelo *New Keynesian* tradicional com dois sectores (sectores de bens duráveis e bens não duráveis) gera co-movimento sectorial negativo e neutralidade da moeda ou falta de co-movimento sectorial. Estes resultados vão contra o senso comum e a evidência de VAR: (i) os produtos agregados e sectoriais movem na mesma direção, e (ii) o preço e o produto do sector durável reagem mais do que os do sector não durável. Neste ensaio, nós reconciliamos o modelo *New Keynesian* com a evidência VAR ao introduzir rigidez real nos salários.

No terceiro ensaio, eu e o Alper Çenesiz, focámo-nos no *unemployment volatility puzzle*. O modelo de *search and matching* tem um problema de volatilidade: o desvio padrão do desemprego é 20 vezes inferior ao observado nos dados. Motivados por este puzzle, nós estendemos um modelo de *real business cycle* que permite ajustamentos do trabalho pelas margens intensiva e extensiva. No modelo, os trabalhadores diferem nas suas *skills*, as empresas gostam de variedade de *skills*, e as famílias têm custos para ajustar o número de *skills* no mercado. O

nosso modelo gera desemprego e produto muito voláteis e uma forte propagação de choques.

# Abstract

My thesis is composed of three essays. In the first essay, I am the first to relate luck and employment protection: I study the effects of employment protection on the roles of luck, merit, and effort, in determining income inequality. To this end, I develop a model of heterogeneous workers and firms that meet subject to matching frictions. In the model, an increase in employment protection strengthens the role of luck and weakens the role of merit in generating income inequality.

In the second essay, Alper Çenesiz and I study the effects of monetary policy on aggregate and sectoral outputs. The standard two sector (with durable and nondurable goods sectors) New Keynesian model generates either negative sectoral comovement and aggregate neutrality or no sectoral comovement. These results are at odds with conventional wisdom and VAR evidence: (i) aggregate and sectoral outputs move together, and (ii) price and output of the durable sector react more strongly than those of the nondurable sector. In this essay, we reconcile the standard two sector New Keynesian model with VAR evidence by introducing real wage rigidities.

In the third essay, Alper Çenesiz and I focus on the unemployment volatility puzzle. The search and matching model, the workhorse model of the labor market, has a volatility problem: The standard deviation of unemployment is about 20 times smaller than that in data. Motivated by this puzzle, we extend a real business cycle model to allow for both intensive and extensive margins of labor in a novel way. In the model, workers differ in their skills, firms love variety of skills, and households incur in costs for adjusting the number of skills in the market. Our model generates highly volatile unemployment, highly volatile output,

and a strong propagation mechanism for output and other relevant macroeconomic variables.

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# **Chapter 1**

## **Luck and Employment Protection: The Role of Heterogeneous Job Quality**

### **1.1 Introduction**

How are luck and employment protection related? For a set of OECD countries, Figure 1.1 plots the OECD employment protection index against the percentage of respondents in the World Values Survey who believe that luck determines income. It suggests that countries with high employment protection tend to have a higher percentage of people who believe that luck determines income. In this chapter, I build a search and matching model in which an increase in employment protection strengthens the role of luck and weakens the role of merit.

Figure 1.1: Luck and employment protection.



*Note:* The figure shows the correlation between the percentage of people who believe that luck determines income and employment protection for a set of OECD countries. The percentage of people who believes that luck determines incomes is constructed from the World Values Survey. In this survey, respondents are asked to grade their views over the importance of luck to determine income in a scale from 1 to 10. 1 is interpreted as “In the long run, hard work usually brings a better life”, and 10 is interpreted as “Hard work doesn’t generally bring success - it’s more a matter of luck and connections.” I take the average of respondents’ answers from 1981 to 2008, and following Alesina et al. (2001), I rescale the answers as a binary variable where respondents that answer 5 or above are considered to believe that luck determines income. Employment protection is the index built by the OECD for “Protection of permanent workers against individual and collective dismissals” in 2013.

The literature on employment protection focuses on the effects of employment protection on labor market flows, unemployment, and efficiency. This literature agrees that employment protection decreases labor market flows because employment protection discourages both firing and hiring. But it does not agree on the effect of employment protection on unemployment, which is clear in the empirical literature surveyed by Addison and Teixeira (2003) and in the theoretical literature surveyed by Ljungqvist (2002).<sup>1</sup> Employment protection also has an ambiguous effect on efficiency. On the one hand, employment protection improves efficiency because it motivates the accumulation of firm-specific human capital (Belot et al., 2007). On the other hand, employment protection deteriorates efficiency because it promotes capital deepening (Autor et al., 2007), decreases workers' reallocation (Bassanini et al. 2009, Petrin and Sivadasan 2013, and Samaniego 2006), and increases absenteeism (Ichino and Riphahn 2005, Jacob 2010, and Riphahn 2004).

The literature on employment protection disregards the relationship between luck and employment protection. Yet, this relationship matters for agents' incentives: If luck solely determines income, agents are discouraged to exert effort and to develop skills. For that matter, in this chapter, I extend the Diamond-Mortensen-Pissarides model to study workers' income inequality as a result of unequal luck, merit, and effort.

In the model, firms are heterogeneous. As in Acemoglu (2001), firms decide to either be good or bad by opening good or bad vacancies. A good firm is more productive and pays higher wages than a bad firm. And unlike bad firms, good firms fire workers with productivity below an endogenous cutoff. Workers are also heterogeneous: They are of high or low skill and they draw their productivity from a skill-specific probability distribution when they meet a firm.

Workers' income inequality results from unequal luck, merit, and effort. Luck relates to any labor market outcome that is independent of workers' merit and effort. In the model, the probabilities that an unemployed worker finds a good job or

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<sup>1</sup>See also Boeri (2011) for an additional survey, and Postel-Vinay and Turon (2014) and Samaniego (2006) for recent theoretical contributions with conflicting results for unemployment.

bad job are independent of workers' merit and effort, and thus, result solely from luck: An unemployed low skill worker has the same probabilities to find a good or a bad job as an unemployed high skill worker.

Merit relates to the effect of skill on labor market outcomes: Because of the productivity difference between high skill and low skill workers, a high skill worker has a higher probability to stay employed in good and better paying jobs than a low skill worker.<sup>2</sup>

Effort relates to the effect of search on-the-job on labor market outcomes. And it can also be interpreted as a means to compensate for the lack of luck. In the model, workers employed in bad firms can search on-the-job to try to move to good firms. Therefore, workers that were not lucky enough to find good jobs when unemployed can exert effort to have the opportunity to move from bad to good jobs.

Employment protection –in the form of firing costs in the model– reshapes the importance of luck, merit, and effort in generating income inequality. An increase in firing costs reduces the value of good vacancies: Good firms are forced to retain less productive workers. Bad firms, on the other hand, benefit from less competition by good firms. These two effects decrease the proportion of good vacancies: *an increase in firing costs deteriorates the average quality of jobs.*

Because firing costs deteriorate the average quality of jobs, workers find less good jobs. This strengthens the role of luck in generating income inequality between high skill workers: An increase in firing costs increases inequality between high skill workers who (are lucky and) find good jobs and those who only find bad jobs or stay unemployed. It also discourages effort to search on-the-job, further exacerbating income inequality.

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<sup>2</sup>Although the distinction between luck and merit can be confusing (e.g. a worker may become more skilled because of its parents ability to send him to better schools), in the model I assume that skill is only a matter of merit.



An increase in firing costs also weakens the role of merit in generating income inequality. An increase in firing costs induces good firms to retain a higher proportion of low skill workers. This implies that low skill workers are more likely to stay employed in good firms, therefore decreasing inequalities generated by unequal merit. *An increase in firing costs strengthens the role of luck and weakens the role of merit.*

The results of my chapter differ from the typical insider-outsider theory. An increase in firing costs is expected to increase the income of insiders (employed and lucky) relative to outsiders (unemployed and unlucky) because firing costs make it harder for an unemployed worker to find a job. My model, however, generates richer results because it features various sources of heterogeneity. First, I study the effects of firing costs in the income inequality of two types of insiders rather than one (workers employed in good firms and workers employed in bad firms). Second, I also study the effects of firing costs on income inequality between workers of unequal skill. And third, I propose a general perspective on the sources of income inequality: I relate income inequality with three sources – Luck, merit, and effort.

Alesina and Angeletos (2005) also relate luck with institutions. They, however, focus on the relationship between luck and the size of the welfare state, whereas I focus on the relationship between luck and employment protection. Alesina and Angeletos show that identical economies converge to different equilibria depending on their agents' perception of the role of luck in determining income. They also show that the perception of the role of luck is self-fulfilling. If agents believe that luck determines income, they prefer a large government with large redistributive policies. But large redistribution discourages effort, and thus, agents prefer to rely on their luck than to work harder and face higher taxes. Because agents believe that luck determines income, they develop institutions that promote luck instead of merit. The opposite happens if agents believe that merit determines income. In that case, redistribution is smaller and agents have incentives to work harder as most of the additional income goes to themselves. Because agents believe that merit determines income, they develop institutions that

promote merit instead of luck.

The remainder of the chapter is structured as follows. In Section 1.2, I describe the model. In Section 1.3, I show how I calibrate the model. In Section 1.4, I discuss the results of the model: I discuss the effects of employment protection on income inequality, worker flows, unemployment, and output. And in Section 1.5, I conclude.

## 1.2 The Model

### 1.2.1 An overview

The model extends the Diamond-Mortensen-Pissarides model by featuring firms' and workers' heterogeneity. I follow Acemoglu (2001) and assume that firms can be good (high productivity) or bad (low productivity). A firm is good if it opens a good vacancy at cost  $\kappa_g$  and a firm is bad if it opens a bad vacancy at cost  $\kappa_b$ .

Workers are of high or low skill. For each skill type, there is a unit interval of workers: A fraction  $u^h$  ( $u^l$ ) of high (low) skill workers is unemployed and a fraction  $1 - u^h$  ( $1 - u^l$ ) is employed. Prior to the match, workers of the same skill are identically-productive. But after the match, their productivity differs, as in Jovanovic (1979). After matching with a firm, workers draw their productivity from a skill-specific uniform probability distribution:  $[\underline{a}, \bar{a}]$  if low skill;  $[\underline{a} + a^h, \bar{a} + a^h]$  if high skill; where  $a^h$  is the average productivity difference between a high skill and a low skill worker. In this labor market, firms open vacancies, and workers search for jobs while unemployed and while employed in bad firms.

Unemployed workers devote all their time, normalized to 1, to search for jobs. Following Albrecht and Vroman (2002), I assume that workers and firms are pooled in a single matching function: The job filling probability is independent of firm's productivity, as well as the job finding probability per unit of search effort is independent of worker's skill. This implies that finding a job –and finding

a good or a bad job—is a merely a matter of luck. Merit differences does not imply different probabilities.

The matching function is concave, increasing, linearly homogeneous, and is given by:

$$M = \mu (u^h + u^l + (1 - \delta)e^h n^{bh} + (1 - \delta)e^l n^{bl})^\eta (v^b + v^g)^{1-\eta}, \quad (1.1)$$

where  $e^h$  ( $e^l$ ) is the effort of each high (low) skill worker to search on-the-job;  $n^{bh}$  ( $n^{bl}$ ) is the number of high (low) skill workers employed in bad firms, and thus, searching on-the-job;  $v^g$  ( $v^b$ ) is the number of vacancies opened by good (bad) firms;  $\mu$  is a scale parameter;  $\delta$  is the exogenous job destruction probability; and  $\eta$  is the elasticity of the matching function with respect to total search effort. I define the job filling probability as

$$\mu(\theta) \equiv \mu\theta^{-\eta}, \quad (1.2)$$

where  $\theta \equiv \frac{v^b + v^g}{(u^h + u^l + e^h n^{bh} + e^l n^{bl})}$  is the vacancy to total search effort ratio. The job finding probability per unit of effort is simply  $f(\theta) \equiv \mu(\theta)\theta$ .

As in Hall (2005b), I assume that real wages are rigid. I also assume that real wages only depend on firm's productivity, and that good firms pay higher wages than bad firms. The two wages in the economy guarantee that all workers are willing to work for all firms. They also guarantee that good firms fire the workers with productivity below an endogenous cutoff  $a^*$  while bad firms have no incentives to fire their workers.<sup>3</sup>

Because of the average productivity difference between high skill and low skill workers, high skill workers are less likely fired. This implies that high skill workers keep a higher proportion of good and better paying jobs than low skill workers, leading to unequal income because of unequal merit. To simplify the model, I further assume that, independent of his productivity draw, a high skill

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<sup>3</sup>With the standard Nash Bargain for wages, I could not consider a case where good firms fire some of their workers, while bad firms do not. For more details, see Section 1.2.4.

worker always produces more than his wage; and thus, only low skill workers are fired:  $a^* \in (\underline{a}, \underline{a} + a^h)$ .

In the model, the sequence of events is as follows: (i) production takes place; (ii) a fraction  $\delta$  of employed workers exogenously lose their jobs and start searching in the following period;<sup>4</sup> (iii) unemployed workers and workers employed in bad firms search for jobs; (iv) luck determines which workers find good jobs, bad jobs, or stay unemployed; (v) firms hire all the workers they are matched with; (vi) each matched worker draws his productivity from his respective probability distribution; (vii) the firm knows the productivity of the worker it was matched with; (viii) merit differences imply that in case a low skill worker draws a productivity below  $a^*$ , good firms pay a sunk waste firing cost,  $\tau$ , to fire him; (ix) the workers that are fired restart searching in the following period.

### 1.2.2 Workers

Workers discount future at rate  $\beta$ . They earn unemployment benefits  $b$  when unemployed, earn a wage  $w^g$  when employed in good firms, and earn a wage  $w^b$  when employed in bad firms. Workers exogenously lose jobs with a probability  $\delta$ , and find a new job with a probability  $f(\theta)$  per unit of effort. If a worker finds a job, it is a good job with a probability  $z$ , which is the proportion of good vacancies:  $z \equiv \frac{v^g}{v^g + v^b}$ .

As in Krause and Lubik (2006), I assume that workers can search on-the-job to maximize their lifetime income when employed in bad firms. By searching on-the-job, workers have the opportunity to increase their lifetime income by moving to a good job. But they also have a cost, which is measured by the cost function  $\sigma(e^i)^\alpha$ ,  $\alpha > 1$ ,  $i = h, l$ .  $e^h$  ( $e^l$ ) is the effort of a high (low) skill worker, and  $\sigma$  is a scale parameter. To simplify, I assume that workers only move if their job is

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<sup>4</sup>I assume that when a job is destroyed for exogenous reasons, there is no need for the payment of firing costs. This type of job destruction is present to mimic retirement or any other type of job destruction agreed between firm and worker without penalty.

not exogenously destroyed and they are offered a good job that they can keep.<sup>5</sup>

A high skill worker can be in one of three states: He can be employed in a good firm, employed in a bad firm, and unemployed. Let  $E^{gh}$ ,  $E^{bh}$ , and  $U^h$  denote the lifetime income of a high skill worker in each of these states. Then, the lifetime incomes of high skill workers are

$$E^{gh} = w^g + \beta[\delta U^h + (1 - \delta)E^{gh}], \quad (1.3)$$

$$E^{bh} = w^b - \sigma(e^h)^\alpha + \beta\delta U^h + \beta(1 - \delta)[e^h f(\theta)z E^{gh} + (1 - e^h f(\theta)z)E^{bh}], \quad (1.4)$$

$$U^h = b + \beta[f(\theta)(z E^{gh} + (1 - z)E^{bh}) + (1 - f(\theta))U^h]. \quad (1.5)$$

When employed in good firms, a high skill worker earns a wage  $w^g$ , and loses his job with a probability  $\delta$ . When employed in bad firms, he earns a wage  $w^b$ , and loses his job with a probability  $\delta$ , but he also searches on-the-job. If he does not lose his job, he has a probability  $e^h f(\theta)z$  to move to a good job, and a probability  $1 - e^h f(\theta)z$  to stay in the bad job. When unemployed, a high skill worker earns unemployment benefits  $b$ , and has a probability  $f(\theta)$  to find a job and  $1 - f(\theta)$  to stay unemployed. If he finds a job, it is a good job with a probability  $z$  and a bad job with a probability  $1 - z$ .

High skill workers choose the effort to search on-the-job,  $e^h$ , to maximize  $E^{bh}$ . Taking wages and the probability of finding a good job per unit of effort as given, the optimal effort satisfies

$$\sigma\alpha(e^h)^{\alpha-1} = \beta(1 - \delta)f(\theta)z(E^{gh} - E^{bh}), \quad (1.6)$$

where  $E^{bh}$  is evaluated at optimal effort. Effort increases with the probability of finding a good job and the value gain by moving from a bad to a good job.

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<sup>5</sup>Krause and Lubik (2006) also assumes that the new job is subject to a job destruction probability even before the worker starts working in the new job. This is equivalent to assuming that the worker only moves if their current job is not exogenously destroyed.

Replacing this condition in Eq. 1.4 implies

$$E^{bh} = w^b + \sigma(e^h)^\alpha(\alpha - 1) + \beta[\delta U^h + (1 - \delta)E^{bh}]. \quad (1.7)$$

As  $\alpha > 1$ , a high skill worker increases his lifetime income when employed in bad firms by searching on-the-job. It is, thus, a means to compensate for the lack of luck when unemployed: A high skill worker employed in a good firm can exert effort to search on-the-job to decrease the inequality between those who find good and bad jobs.

High skill workers are, prior to the match, identically productive and choose the same effort to search on-the-job when employed in bad firms. Thus, any inequality between them does not result from merit or effort differences, but rather from luck differences.

Low skill workers differ from high skill workers in the ability to keep good jobs: While a high skill worker always draws a productivity above the cutoff, a low skill worker draws a productivity above the cutoff with a probability  $\frac{\bar{a}-a^*}{\bar{a}-\underline{a}} < 1$ . Let  $E^{gl}$ ,  $E^{bl}$ , and  $U^l$  denote the lifetime income of a low skill worker employed in a good firm, employed in a bad firm, and unemployed. These values are

$$E^{gl} = w^g + \beta[\delta U^l + (1 - \delta)E^{gl}], \quad (1.8)$$

$$E^{bl} = w^b - \sigma(e^l)^\alpha + \beta\delta U^l + \beta(1 - \delta) \left[ e^l f(\theta) z \frac{\bar{a} - a^*}{\bar{a} - \underline{a}} E^{gl} + \left( 1 - e^l f(\theta) z \frac{\bar{a} - a^*}{\bar{a} - \underline{a}} \right) E^{bl} \right], \quad (1.9)$$

$$U^l = b + \beta \left[ f(\theta) \left( z \frac{\bar{a} - a^*}{\bar{a} - \underline{a}} E^{gl} + (1 - z) E^{bl} \right) + (1 - f(\theta)) U^l \right]. \quad (1.10)$$

Low skill workers choose the effort to search on-the-job,  $e^l$ , to maximize  $E^{bl}$ . Taking wages and the probability of finding and keeping a good job per unit of

effort as given, the optimal effort satisfies

$$\sigma\alpha(e^l)^{\alpha-1} = \beta(1-\delta)f(\theta)z\frac{\bar{a}-a^*}{\bar{a}-\underline{a}}(E^{gl}-E^{bl}), \quad (1.11)$$

where  $E^{bl}$  is evaluated at optimal effort. As low skill workers have smaller chances to keep good jobs, they exert less effort than high skill workers to search on-the-job; but the smaller is the cutoff (closer to  $\underline{a}$ ), the smaller is this effort difference. Replacing this condition in Eq. 1.9 implies

$$E^{bl} = w^b + \sigma(e^l)^\alpha(\alpha-1) + \beta[\delta U^l + (1-\delta)E^{bl}]. \quad (1.12)$$

### 1.2.3 Firms

As standard in the search and matching literature, each firm employs one worker. In the model, firms decide to either be good or bad by opening good or bad vacancies. The decision to be good or bad depends on the value of the two types of vacancies. A good firm has productivity  $y^g$ , which is greater than the productivity  $y^b$  of a bad firm. A good firm also fills a vacancy much faster than a bad firm because it attracts workers employed in bad firms. But a good firm pays higher wages and may meet with workers who produce less than their wage. The choice of vacancy type depends on balancing these effects together with the costs to keep a vacancy open:  $\kappa_g$  if a good vacancy, and  $\kappa_b$  if a bad vacancy. In equilibrium, however, firms are indifferent between opening good or bad vacancies. As standard in the literature, the value of a vacancy equals the marginal cost to open a vacancy, which is 0.

The output of a firm is the product of firm's productivity with worker's productivity. This implies that the value of a good job filled by a worker who draws a productivity  $a$  is

$$J^g(a) = y^g a - w^g + \beta[(1-\delta)J^g(a) + \delta V^g], \quad (1.13)$$

where  $V^g$  is the value of a good vacancy. The value of a job is the sum of (i) the difference between the match's output and wage and (ii) the continuation value of

the job. With a probability  $\delta$  the job is exogenously destroyed and the firm has a new vacancy. The value of a good vacancy is

$$V^g = -\kappa^g + \mu(\theta)\beta [(p_u^h + p_e^h)J^{gh} + (1 - p_u^h - p_e^h)J^{gl}] + [1 - \mu(\theta)]\beta V^g. \quad (1.14)$$

This value (as for the value of any other vacancy) depends on the cost to keep the vacancy open ( $\kappa^g$ ), on-the-job filling probability, on the average value of a job, and on the continuation value of the vacancy. The job filling probability is simply  $\mu(\theta)$ . The average value of a job, on the other hand, is more complex: It depends on the probability to find a high skill or a low skill worker, and on their respective value for a good firm. A good firm meets with a high skill worker with a probability  $p_u^h + p_e^h$  (where  $p_u^h$  is the proportion of search effort made by unemployed high skill workers and  $p_e^h$  is the proportion of search effort made by high skill workers employed in bad firms). And meets with a low skill worker with a probability  $1 - p_u^h - p_e^h$ . Regarding the values of the two types of workers,  $J^{gh}$  for high skill and  $J^{gl}$  for low skill, they are given by

$$J^{gh} = y^g \left( \underline{a} + a^h + \frac{\bar{a} - \underline{a}}{2} \right) - w^g + \beta [(1 - \delta)J^{gh} + \delta V^g], \quad (1.15)$$

$$J^{gl} = \frac{\bar{a} - a^*}{\bar{a} - \underline{a}} \left[ y^g \left( a^* + \frac{\bar{a} - a^*}{2} \right) - w^g + \beta ((1 - \delta)J^{gl} + \delta V^g) \right] + \frac{a^* - \underline{a}}{\bar{a} + \underline{a}} (V^g - \tau). \quad (1.16)$$

The value of a good job filled by a high skill worker is obtained when  $a$  in Eq. 1.13 is replaced by the average productivity of a high skill worker. The value of a good job filled by a low skill worker is not so simple. There is a probability  $\frac{\bar{a} - a^*}{\bar{a} - \underline{a}}$  that a low skill worker draws a productivity above or equal  $a^*$ . In this case, the average value of a job is now obtained when  $a$  in Eq. 1.13 is replaced by the average productivity of low skill workers who draw a productivity above the cutoff. But a low skill worker has also a probability  $\frac{a^* - \underline{a}}{\bar{a} - \underline{a}}$  of drawing a productivity below the cutoff. In this case, the worker is fired, and the firm pays the firing cost  $\tau$  and has



a new vacancy.

The value of a worker for a bad firm depends on the ability of the worker to move from bad to good jobs. This ability is not the same for a high skill worker and for a low skill worker: High skill workers exert more effort than low skill workers to search on-the-job because they have a higher probability to stay employed in good firms; furthermore, low skill workers do not retain all the good jobs they find while high skill workers do retain. The value of a bad job filled by a high skill or low skill worker must, then, be adjusted:

$$J^{bh}(a) = y^b a - w^b + \beta \left[ (1 - \delta)(1 - e^h f(\theta)z) J^{bh}(a) + (1 - (1 - \delta)(1 - e^h f(\theta)z)) V^b \right], \quad (1.17)$$

$$J^{bl}(a) = y^b a - w^b + \beta(1 - \delta) \left( 1 - e^l \frac{\bar{a} - a^*}{\bar{a} - \underline{a}} f(\theta)z \right) J^{bl}(a) + \beta \left( 1 - (1 - \delta) \left( 1 - e^l \frac{\bar{a} - a^*}{\bar{a} - \underline{a}} f(\theta)z \right) \right) V^b, \quad (1.18)$$

where  $V^b$  is the value of a bad vacancy. The intuition behind these equations is the same as for the value of good jobs: These values increase with the output of the match, decrease with wages, and increase with the continuation value of the job. But the continuation values of good and bad jobs differ. A bad firm keeps a high skill worker with a probability  $(1 - \delta)(1 - e^h f(\theta)z)$  and a low skill worker with a probability  $(1 - \delta) \left( 1 - e^l \frac{\bar{a} - a^*}{\bar{a} - \underline{a}} f(\theta)z \right)$ .<sup>6</sup> As for the value of a bad vacancy, it is

$$V^b = -\kappa^b + \mu(\theta) [p_u^h J^{bh} + p_u^l J^{bl}] + \beta [1 - \mu(\theta)(p_u^h + p_u^l)] V^b, \quad (1.19)$$

where  $p_u^l$  is the proportion of search effort made by unemployed low skill workers. Bad firms keep all the workers they meet, and only attract unemployed workers.

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<sup>6</sup>Jobs are exogenously destroyed with a probability  $\delta$ . If they are not destroyed, a high skill worker has a probability  $e^h f(\theta)z$  to move to a good job, while a low skill worker has a probability  $\left( e^l \frac{\bar{a} - a^*}{\bar{a} - \underline{a}} f(\theta)z \right)$ .

The value of a bad job is again the weighted average of the values of filling the vacancy with a high skill and a low skill worker. With a probability  $\mu(\theta)p_u^h$ , bad firms fill a job with an unemployed high skill worker, and on average the value of that job is  $J^{bh}$  (which is obtained when  $a$  in Eq.1.17 is replaced by the average productivity draw of a high skill worker); With a probability  $\mu(\theta)p_u^l$  bad firms fill a job with an unemployed low skill worker, and on average the value of that job is  $J^{bl}$  (which is given by Eq.1.18 with  $a$  replaced by the average productivity draw of a low skill worker).

As mentioned above, firms open vacancies until  $V^g = V^b = 0$  because of the free-entry condition. Then, solving Eqs. 1.14 and 1.19 imply the following equilibrium conditions:

$$\begin{aligned} \kappa^g = & \mu(\theta)\beta \left[ (p_u^h + p_e^h) \frac{y^g[\underline{a} + a^h + (\bar{a} - \underline{a})/2] - w^g}{1 - (1 - \delta)\beta} \right] + \\ & \mu(\theta)\beta \left[ (1 - p_u^h - p_e^h) \left( \frac{\bar{a} - a^*}{\bar{a} - \underline{a}} \frac{y^g[a^* + (\bar{a} - a^*)/2] - w^g}{1 - (1 - \delta)\beta} - \frac{a^* - \underline{a}}{\bar{a} - \underline{a}} \tau \right) \right], \end{aligned} \quad (1.20)$$

$$\begin{aligned} \kappa^b = & \mu(\theta)\beta p_u^h \frac{y^b[\underline{a} + a^h + (\bar{a} - \underline{a})/2] - w^b}{1 - (1 - \delta)(1 - e^h f(\theta)z)\beta} \\ & + \mu(\theta)\beta p_u^l \frac{y^b[\underline{a} + (\bar{a} - \underline{a})/2] - w^b}{1 - (1 - \delta)(1 - e^l \frac{\bar{a} - a^*}{\bar{a} - \underline{a}} f(\theta)z)\beta}, \end{aligned} \quad (1.21)$$

where I have also used Eqs. 1.15-1.18.

## 1.2.4 Cutoff

A good firm keeps a worker if it is better off keeping him than firing him: The firm contrasts the value of keeping the worker with the value of firing him. If positive, the firm keeps the worker; if negative, the firm fires the worker. The value of keeping a worker who draws a productivity  $a$  is given by the value of a good job,  $J^g(a)$ , while the value of firing is given by  $V^g - \tau$ . As  $V^g = 0$ , the firm is indifferent between keeping or firing a worker in the case that  $J^g(a) + \tau = 0$ .

Making use of Eq. 1.13 and solving for  $a$ , I find the cutoff equation:

$$a^* = \frac{w^g - \tau(1 - (1 - \delta)\beta)}{y^g}. \quad (1.22)$$

In case there are no firing costs,  $\tau = 0$ , good firms keep workers that produce at least as much as their wage. But if firing costs exist,  $\tau > 0$ , good firms are forced to retain workers who produce less than their wage.

### 1.2.5 Wages

To close the model, I do not assume that wages are determined by Nash Bargain. With Nash Bargain, the higher is the surplus of the match, the higher are the incentives to hire the worker. The surplus of the match depends positively on the output of the match, and thus, is on average higher in good firms. This implies that with Nash Bargain, bad firms are more likely to fire a worker than good firms.

In this chapter, however, I wish to study the effects of employment protection in a context where good firms demand more productive workers than bad firms. In particular, I wish to study the effects of employment protection in a context where the following conditions are met: (i) good firms only wish to retain the most productive low skill workers they meet; (ii) good firms wish to keep all high skill workers they meet; and (iii) bad firms wish to keep all workers they meet. The simplest way to achieve this is to follow Hall (2005b) and Shimer (2012b) and assume that wages are rigid. Thus, I assume that the two wages in the economy—the one set by good firms and the one set by bad firms—are rigid and satisfy conditions (i)-(iii). These wages, however, must also motivate workers to find jobs. In particular, they must also satisfy the following conditions: (iv) all workers are willing to work for bad firms, and (v) all workers are willing to work for good firms. In the remainder of this subsection, I show how to set  $w^g$  and  $w^b$  such that they satisfy conditions (i)-(v).<sup>7</sup>

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<sup>7</sup>These conditions are an extension of the conditions in Hall (2005b) and Shimer (2012b). In these papers, wages are set such that workers are willing to work, and firms are willing to hire. In this chapter, however, firms' and workers' heterogeneity requires further conditions.

The cutoff equation, Eq. 1.22, can be rearranged as

$$w^g = y^g a^* + \tau(1 - (1 - \delta)\beta). \quad (1.23)$$

The first condition (good firms only wish to retain the most productive low skill workers they meet), is equivalent to impose that the cutoff is in the distribution of productivity draws of low skill workers,  $a^* \in (\underline{a}, \bar{a})$ . Applying this to Eq. 1.23 implies that

$$w^g \in (y^g \underline{a} + \tau(1 - (1 - \delta)\beta), y^g \bar{a} + \tau(1 - (1 - \delta)\beta)).$$

To satisfy the second condition (good firms wish to keep all high skill workers they meet), I impose that whatever the productivity draw of a high skill worker, the value (for a good firm) of keeping him exceeds the value of firing:  $w^g$  must be such that for the high skill worker who draws the smallest productivity draw,  $\underline{a} + a^h$ , the value of keeping him exceeds the value of firing. Applying this to Eq. 1.23 implies that

$$w^g < y^g(\underline{a} + a^h) + \tau(1 - (1 - \delta)\beta).$$

To satisfy the third condition (bad firms wish to keep all workers they meet), I impose that independent of the productivity draw of a worker, the value (for a bad firm) of keeping him exceeds the value of firing. The workers with the lowest productivity are low skill workers who draw the productivity  $\underline{a}$ . Using an analogous equation to Eq. 1.23, where  $w^g$  is replaced by  $w^b$  and  $y^g$  is replaced by  $y^b$ , I find that the third condition is satisfied when

$$w^b < y^b \underline{a} + \tau(1 - (1 - \delta)\beta).$$

To satisfy the forth and fifth conditions, I start by imposing that  $w^b > b$  and  $w^g > b$ . I divide the forth condition (all workers are willing to work for bad firms), in two parts: Bad firms must attract low skill workers,  $E^{bl} - U^l > 0$ , and high skill workers,  $E^{bh} - U^h > 0$ . The value of being unemployed for a high skill worker is greater than the value of being an unemployed for a low skill worker because high skill workers have better odds of keeping good jobs. Thus, if  $w^b$  satisfies  $E^{bh} - U^h > 0$ , it also satisfies  $E^{bl} - U^l > 0$ . Because of this, I focus on satisfying

the condition that  $E^{bh} - U^h > 0$ . Using Eqs. 1.3, 1.5, and 1.7 I find that

$$w^b > -\sigma(e^h)^\alpha(\alpha - 1) + \frac{(1 - \beta)b(1 - (1 - \delta)\beta) + (1 - \beta)\beta f(\theta)zw^g}{1 - (1 - \delta)\beta - \beta^2\delta f(\theta) - \beta(1 - (1 - \delta)\beta)(1 - f(\theta)) - (1 - \beta)\beta f(\theta)(1 - z)} \quad (1.24)$$

is necessary to satisfy  $E^{bh} - U^h > 0$ .

The fifth condition (all workers are willing to work for good firms), is satisfied in combination with the other conditions. Because any worker produces on average more in a good firm than in a bad firm ( $y^g > y^b$ ), conditions (i) and (iii) imply that  $w^g > w^b$ . Thus, if conditions (i), (iii), and (iv) are satisfied, then condition (v) is also satisfied.

### 1.2.6 Equilibrium

I now show the equilibrium conditions. The high and low skill unemployment rates are

$$u^h \equiv \frac{\delta}{\delta + f(\theta)}, \quad (1.25)$$

$$u^l \equiv \frac{\delta}{\delta + zf(\theta)\frac{\bar{a}-a^*}{\bar{a}-a} + (1 - z)f(\theta)}. \quad (1.26)$$

These unemployment rates differ because of the difference in the likelihood of keeping good jobs: High skill workers only become unemployed if their jobs are exogenously destroyed, while low skill workers also become unemployed if they draw a productivity below  $a^*$  in good firms.

All workers employed in bad firms,  $n^{bh} + n^{bl}$ , search on-the-job. To find total search effort, I start by finding  $n^{bh}$  and  $n^{bl}$ . Each period, there are  $u^h$  high skill unemployed workers and  $u^l$  low skill unemployed workers. From those, every period a fraction  $f(\theta)(1 - z)$  find bad jobs. High skill workers whose jobs are not exogenously destroyed have a probability  $f(\theta)z$  of finding a good job per unit of effort, and choose effort  $e^h$ : A fraction  $(1 - \delta)f(\theta)ze^h$  of high skill workers searching on-the-job move to a good job, and a fraction  $(1 - \delta)(1 - f(\theta)ze^h)$

do not move. Thus, the number of high skill workers employed in bad firms and searching on-the-job is

$$n^{bh} = \frac{u^h f(\theta)(1-z)}{1 - (1-\delta)(1-f(\theta)ze^h)}. \quad (1.27)$$

The low skill workers whose jobs are not exogenously destroyed have a probability  $f(\theta)z\frac{\bar{a}-a^*}{\bar{a}-\underline{a}}$  of finding a good job per unit of effort, and choose effort  $e^l$ : A fraction  $(1-\delta)f(\theta)z\frac{\bar{a}-a^*}{\bar{a}-\underline{a}}e^l$  of high skill workers searching on-the-job moves to a good job, and a fraction  $(1-\delta)(1-f(\theta)z\frac{\bar{a}-a^*}{\bar{a}-\underline{a}}e^l)$  does not move. Thus, the number of low skill workers employed in bad firms and searching on-the-job is

$$n^{bl} = \frac{u^l f(\theta)(1-z)}{1 - (1-\delta)(1-f(\theta)z\frac{\bar{a}-a^*}{\bar{a}-\underline{a}}e^l)}. \quad (1.28)$$

The proportions of search effort made by unemployed high skill, unemployed low skill, and employed high skill workers are

$$p_u^h = \frac{u^h}{u^h + (1-\delta)e^h n^{bh} + (1-\delta)e^l n^{bl} + u^l}, \quad (1.29)$$

$$p_u^l = \frac{u^l}{u^h + (1-\delta)e^h n^{bh} + (1-\delta)e^l n^{bl} + u^l}, \quad (1.30)$$

$$p_e^h = \frac{(1-\delta)e^h n^{bh}}{u^h + (1-\delta)e^h n^{bh} + (1-\delta)e^l n^{bl} + u^l}. \quad (1.31)$$

As long as wages satisfy the conditions in the previous subsection, an equilibrium in this model is characterized by the set  $(a^*, \theta, z, e^h, e^l)$  that satisfy Eqs. 1.6, 1.11, 1.20, 1.21, and 1.22.<sup>8</sup>

## 1.3 Calibration

The calibration is summarized in Table 3.1. I calibrate the model to the US economy, and each period as a month. In the US economy there are almost no firing

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<sup>8</sup>Before solving the system, I must replace Eqs. 1.3, 1.7, 1.8, 1.12, and 1.25-1.31, in Eqs. 1.6, 1.11, 1.20, 1.21, and 1.22.

Table 1.1: Benchmark Parameters

Discount Rate:	$\beta = 0.996$
Job Destruction Rate:	$\delta = 0.026$
Matching Function Scale Parameter:	$\mu = 0.45$
Matching Function Elasticity Parameter:	$\eta = 0.5$
Unemployment Benefits:	$b = 0$
Elasticity of the on-the-job cost function:	$\alpha = 2$
Scale parameter of the on-the-job cost function:	$\sigma = 0.751$
Productivity of good firms:	$y^g = 1.05$
Productivity of bad firms:	$y^b = 1$
Lowest productivity draw:	$\underline{a} = 1$
Highest productivity draw:	$\bar{a} = 1.25$
High-Low skill average productivity difference:	$a^g = 0.25$
Good firm cost to open a vacancy:	$\kappa^g = 3.068$
Bad firm cost to open a vacancy:	$\kappa^b = 0.776$

costs, and thus, like Samaniego (2006), I set  $\tau = 0$ .<sup>9</sup> I set  $\beta = 0.996$  which implies an annual interest rate of 5%. I set the elasticity of the matching function  $\eta = 0.5$  consistent with the literature surveyed by Petrongolo and Pissarides (2001). As in Shimer (2005), I use  $\mu$  to target a monthly job finding probability,  $f(\theta)$ , equal to 0.45. I also set  $\theta = 1$ , which implies that  $\mu(\theta) = f(\theta) = 0.45$ . Unemployment benefits  $b$ , contrary to models that assume Nash Bargain for wages, do not play an important role in this model because wages are exogenous. Thus, I simply set  $b = 0$ .

Regarding the calibration of the productivity difference –between firms and between labor types– I start by normalizing  $\underline{a}$  and  $y^b$  to 1. Garibaldi and Moen (2010) set a productivity difference between the most and the least productive firms of 8%. In my model, good firms are on average endogenously more productive than bad firms because they only keep low skill workers that draw productivity above the cutoff,  $a^*$ , while bad firms keep all workers. Thus, I assume a smaller exogenous productivity difference and set  $y^g = 1.05$ .

<sup>9</sup>The US economy has the second lowest OECD employment protection index, as is clear in Fig. 1.1.

I assume that the productivity difference between the highest and smallest productivity draw,  $\bar{a} - \underline{a}$ , equals the parameter governing the average productivity difference between high skill and low skill workers,  $a^h$ . As  $a^* < \bar{a}$ , this assumption guarantees that high skill workers are never fired. Albrecht and Vroman (2002) assume that the productivity of high skill jobs is between 20% and 40% higher than the productivity of low skill jobs, and only high skill workers can work in high skill jobs. I start by setting  $a^h = 0.25$ , and later experiment with  $a^h$  ranging from 0.15 to 0.35. By making this experiment, as I keep  $a^*$  constant, a change in  $a^h = \bar{a} - \underline{a}$  changes the proportion of workers that are fired. Finally, I set the cutoff  $a^* = 1.05$ , such that 20% of low skill workers that meet good firms are fired, and later experiment with 1.025 and 1.075.<sup>10</sup>

In reality firms cannot be simply divided between good and bad firms. This makes the proportion of good vacancies in overall vacancies,  $z$ , difficult to target.<sup>11</sup> What makes a firm a good firm? What makes a firm a bad firm? I avoid these questions and start by assuming that half of the vacancies created are good and set  $z = 0.5$ . Later I experiment with  $z = 0.3$  and  $z = 0.7$  to better understand its role in the model. The proportion of jobs exogenously destroyed,  $\delta$ , is used to target the average US unemployment rate in the postwar period, 5.7%.

I use the work of Fallick and Fleischman (2004) on the job-to-job movements in the US to target  $e^h$ . Fallick and Fleischman report that on average a worker that reports to be actively searching on-the-job have an 11% probability to change employers. This is also the number targeted by Moscarini (2001). In my model, this probability is  $e^h(1 - \delta)f(\theta)z$  for high skill workers. But an 11% probability to move between jobs per month implies  $e^h = 0.51$ . This means that employed workers exert half the effort that unemployed workers do to find jobs, which seems

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<sup>10</sup>I calibrate the proportion of workers fired to be small to be more consistent with two assumptions of the model. If the proportion of low skill workers fired is large, it would be more realistic to assume two matching functions (one for high skill workers and one for low skill workers) instead of one. And it would also be more realistic to assume that good firms could reject low skill workers without the need to hire them first. In the latter case, firing costs would have no effect in my model.

<sup>11</sup>Krause and Lubik (2006), for example, assume that the weight of the output of bad firms in total output is 0.4.



unrealistic because of time limitations. Furthermore, recall that high skill workers only move between jobs if they find a good job to simplify the analysis. Thus, if instead workers move between employers if they have an offer from a new employer, the probability of moving between employers would be  $e^h(1 - \delta)f(\theta)$ . In this case,  $e^h = 0.258$ , which seems more realistic.<sup>12</sup> Christensen et al. (2005) estimate an elasticity of the cost function with respect to effort,  $\alpha$ , of 1.84 for the private sector, 1.41 for skilled workers, and 2.3 for unskilled workers.<sup>13</sup> In my benchmark calibration, I set  $\alpha = 2$ , but in the robustness checks, I set  $\alpha = 1.41$  and  $\alpha = 2.3$ . Finally, I set  $\sigma$  to target  $e^h$ , and set  $k^b$  and  $k^g$  to target  $z$  and  $a^*$ .

## 1.4 Results

What are the effects of firing costs? How do luck, merit, and effort contribute to lifetime income inequality under different levels of firing costs?<sup>14</sup> I address these questions by numerically simulating the model.

### 1.4.1 The Effects of Firing Costs

#### Good Firms

Figure 1.2 plots the proportion of good vacancies,  $z = \frac{v^g}{v^g + v^b}$ , and good jobs as a function of  $\tau$ .<sup>15</sup> The proportion of good vacancies falls dramatically with the increase of firing costs. For example, with firing costs equivalent to one and a half months of wages paid by good firms,  $\tau = 1.5w^g$ , the proportion of good vacancies

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<sup>12</sup>I did not use the average of effort of high skill and low skill workers, but in the robustness section, I experiment with  $e^h = 0.5$  and also with  $e^h = 0.15$ . This range encompasses the average effort of high skill and low skill workers.

<sup>13</sup>Their definition of skilled and unskilled workers, however, differ from the high skill and low skill workers in my model.

<sup>14</sup>In the remainder of this chapter, I refer to *lifetime income inequality* as simply *income inequality* for the sake of brevity.

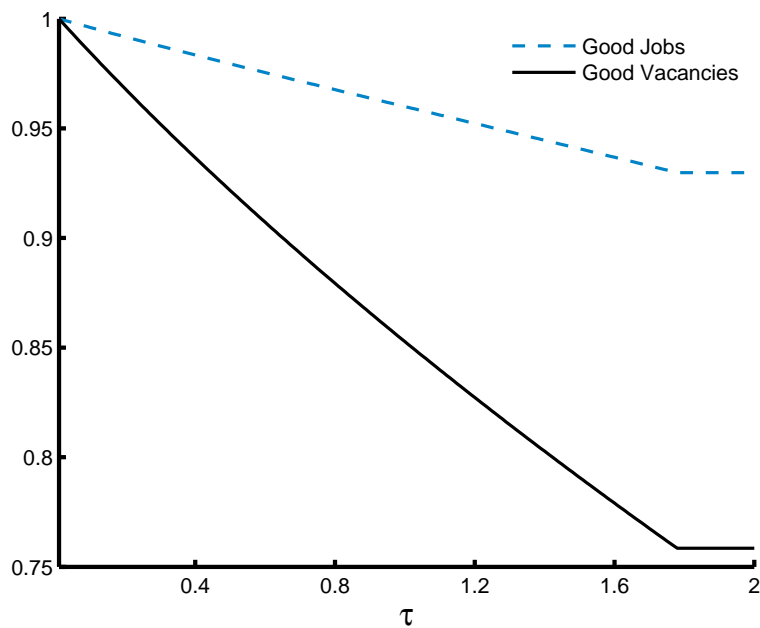
<sup>15</sup>The proportion of good jobs is the mean of the proportion of high skill and low skill workers employed in good firms:  $\left( \frac{1-u^h-n^{bh}}{1-u^h} + \frac{1-u^l-n^{bl}}{1-u^l} \right) / 2$ .

falls by 23%.

The fall of the proportion of good vacancies follows from the fall in their value. As firing costs increase, good firms are forced to retain less productive workers (decreasing the cutoff) and pay the increased cost for the workers they do fire.

On-the-job search propagates the fall of the value of good vacancies. As the proportion of good vacancies falls, both high skill and low skill workers decrease on-the-job search. And because the cutoff decreases, low skill workers decrease their effort to search on-the-job by less than high skill workers (see Panel B of Figure 1.3 and Panel C of 1.4). Thus, good firms not only attract less workers, they attract less productive workers.

Figure 1.2: The effect of firing costs on good jobs and vacancies.



*Note:* The figure shows the proportion of good jobs and good vacancies ( $z$ ), as a function of firing costs,  $\tau$ . Both are relative to the steady-state without firing costs ( $\tau = 0$ ).

Bad firms, on the other hand, benefit from higher firing costs. As they keep all workers, a shift in firing costs has no direct effect. Yet, they benefit from less

competition from good firms (which open less vacancies) and from the fall in the effort to search on-the-job. In the end, *an increase in firing costs deteriorates the average quality of vacancies and jobs.*

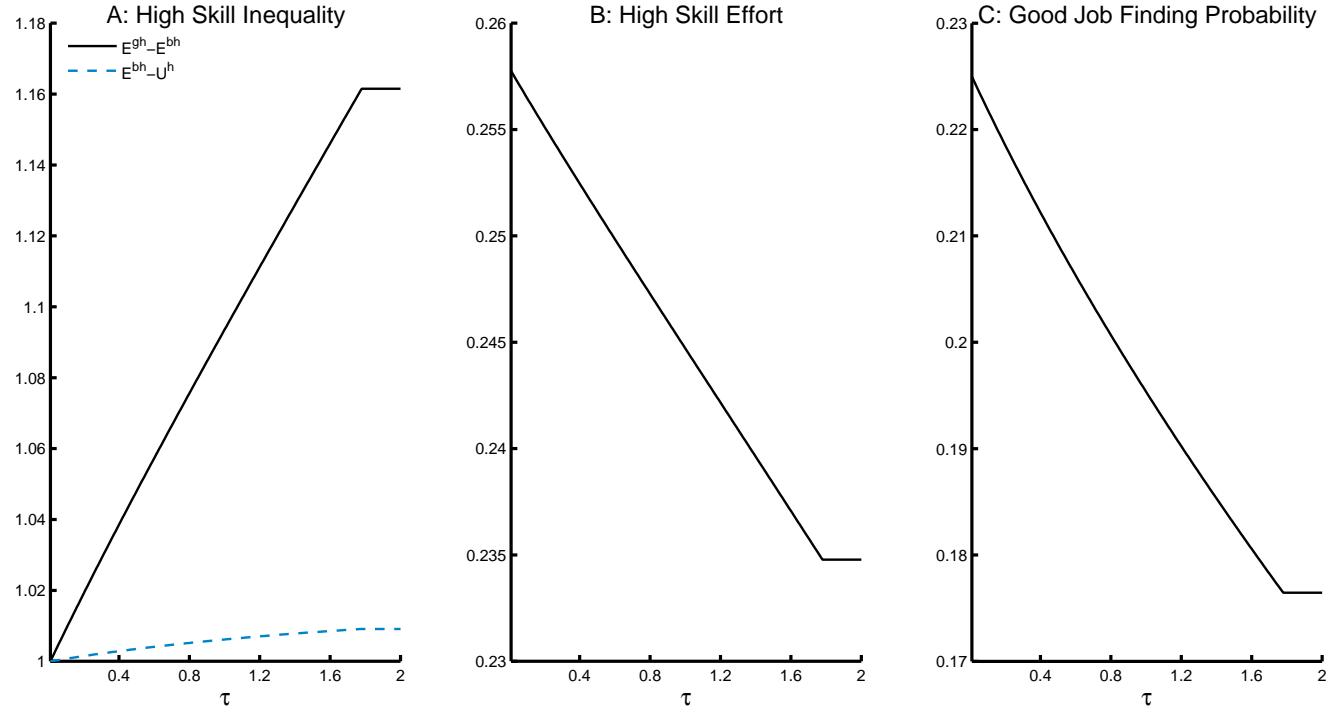
### **Luck, Merit, and Effort**

Figs. 1.3 and 1.4 show how firing costs change the roles of luck, merit, and effort in generating income inequality. Luck relates to any labor market outcome that is independent of worker's merit and effort. High skill workers have the same merit (the same average productivity) and exert the same effort (to search on-the-job). Thus, inequalities between high skill workers result merely from unequal luck.<sup>16</sup> Panel A of Fig. 1.3 shows that income inequality among high skill workers increases with firing costs. As firing costs increase, the good job finding probability decreases. This discourages effort to search on-the-job, thereby accentuating the inequality between those who find good and bad jobs. Furthermore, when unemployed, high skill workers have smaller chances to find goods jobs: Only very lucky high skill workers are able to find good jobs.

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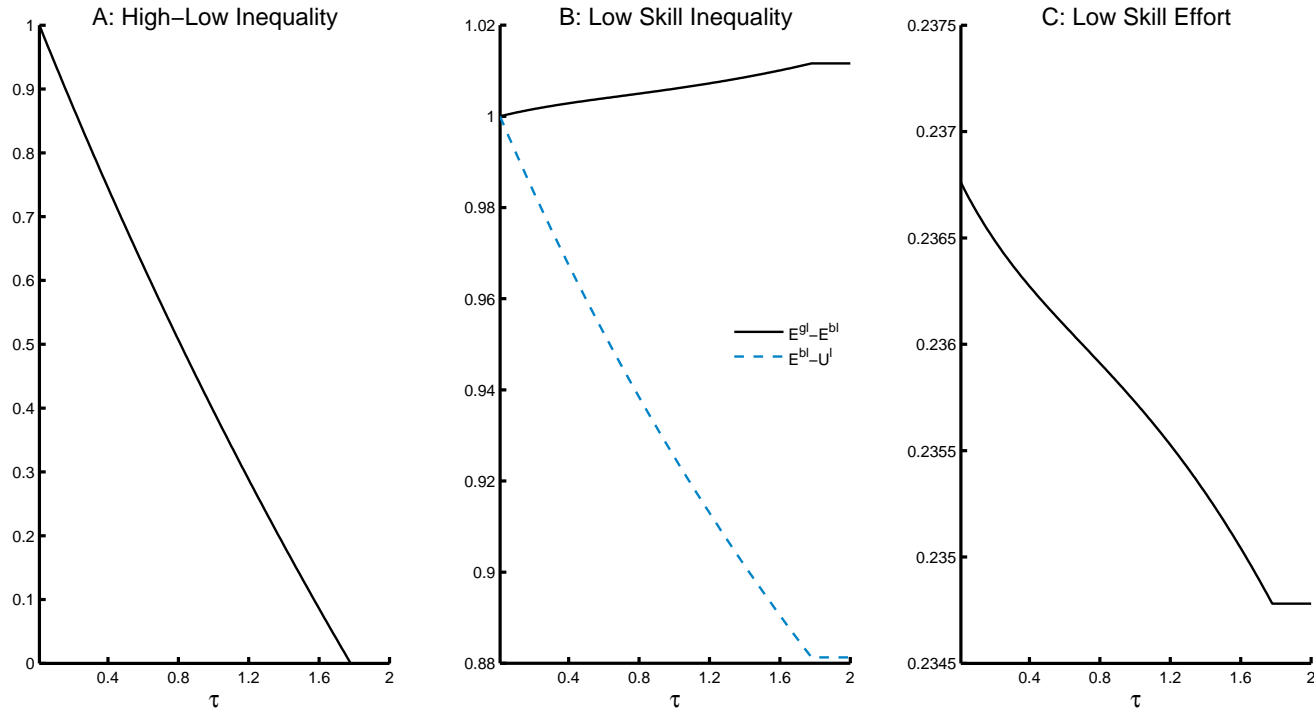
<sup>16</sup>The roles of luck and merit are, however, ambiguous in generating income inequality between low skill workers. Low skill workers only keep a good job if they draw a productivity above the cutoff. If they do not keep a good job is because of a lack of merit: They produce less than their wage. Yet, they are also unlucky if they do not stay employed because they could have drawn a productivity above the cutoff. For that reason, I disregard the effect of firing costs in generating income inequality between low skill workers.

Figure 1.3: The effect of firing costs on inequality, effort, and good job finding probability.



*Note:* The figure shows income inequalities between high skill workers, effort of high skill workers, and good job finding probability as a function of firing costs,  $\tau$ . In Panel A, the figure shows income inequalities relative to the steady-state without firing costs ( $\tau = 0$ ). These income inequalities are only between high skill workers: The figure shows income inequality between high skill workers who find good jobs and those who find bad jobs ( $E^{gh} - E^{bh}$ ) and income inequality between high skill workers who find bad jobs and those who are unemployed ( $E^{bh} - U^h$ ). In Panel B, the figure shows the effort of high skill workers to search on-the-job ( $e^h$ ). In Panel C, the figure shows the good job finding probability per unit of search effort.

Figure 1.4: The effect of firing costs on inequality and effort.



*Note:* The figure shows income inequality between high skill and low skill workers, inequalities between low skill workers, and effort of low skill workers as a function of firing costs,  $\tau$ . In Panels A and B, the figure shows income inequalities relative to the steady-state without firing costs ( $\tau = 0$ ). In Panel A, it shows income inequality between high skill and low skill workers (measured by  $U^h - U^l$ ). In Panel B, it shows income inequality between low skill workers: The figure shows income inequality between low skill workers who keep good jobs and those who keep bad jobs ( $E^{gl} - E^{bl}$ ) and income inequality between low skill workers who keep bad jobs and those who are unemployed ( $E^{bl} - U^l$ ). In Panel C, the figure shows the effort of low skill workers to search on-the-job ( $e^l$ ).

Merit relates to the effect of skill on labor market outcomes. High skill workers are on average more productive than low skill workers. For that matter, high skill workers retain all good and better paying jobs while low skill workers only retain a fraction, allowing high skill workers to attain on average a higher income than low skill workers. An increase in firing costs, however, reduces the effect of skill on labor market outcomes. An increase in firing costs makes good firms retain a higher proportion of low skill workers. This implies that low skill workers become more likely to stay employed in good firms, decreasing income inequality between high and low skill workers (see Panel A of Fig. 1.4). In the extreme, when good firms do not fire any worker, the inequality between high skill and low skill workers ceases to exist.

Effort relates to the effect of search on-the-job on labor market outcomes. Workers employed in bad firms can compensate for their lack of luck (because they only found a bad job) by exerting effort to search on-the-job. Thus, effort is a tool for workers to lessen the relevance of luck in generating income inequality. Panel B of Fig. 1.3 plots the effort of high skill workers to search on-the-job. An increase in firing costs discourages high skill workers to exert effort because it becomes harder to find a good job, exacerbating the effect of luck in generating income inequality.<sup>17</sup>

In the end, *firing costs strengthen the role of luck and weaken the role of merit*. By deteriorating the average quality of jobs, high skill workers are forced to stay employed in bad firms and to rely more on their luck to meet good firms. Furthermore, the inequality generated by unequal merit decreases with firing costs as low skill workers are able to retain a higher proportion of good jobs.

### **Labor Market Flows, Unemployment, and Efficiency**

To compare with the literature on employment protection, I also study labor market flows, unemployment, and efficiency using my model. Panels A and B of Fig.

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<sup>17</sup>Panel C of 1.4 plots the effort of low skill workers to search on-the-job. They also reduce effort, but by less than high skill workers. Although low skill workers also find less good jobs, they keep a higher proportion of those they find because of the lower cutoff.

1.5 plot workers fired and job-to-job movements. As expected, workers fired decrease because good firms retain less productive workers. Job-to-job movements, on the other hand, increase as in Postel-Vinay and Turon (2014). Although, for each worker it is less likely to move from bad to good jobs (less effort and less probability per unit of effort), the number of workers employed in bad firms (willing to search on-the-job) goes up. Furthermore, low skill workers are more likely to stay employed in good firms when firing costs are high. It turns out that the latter two effects are stronger than the former.

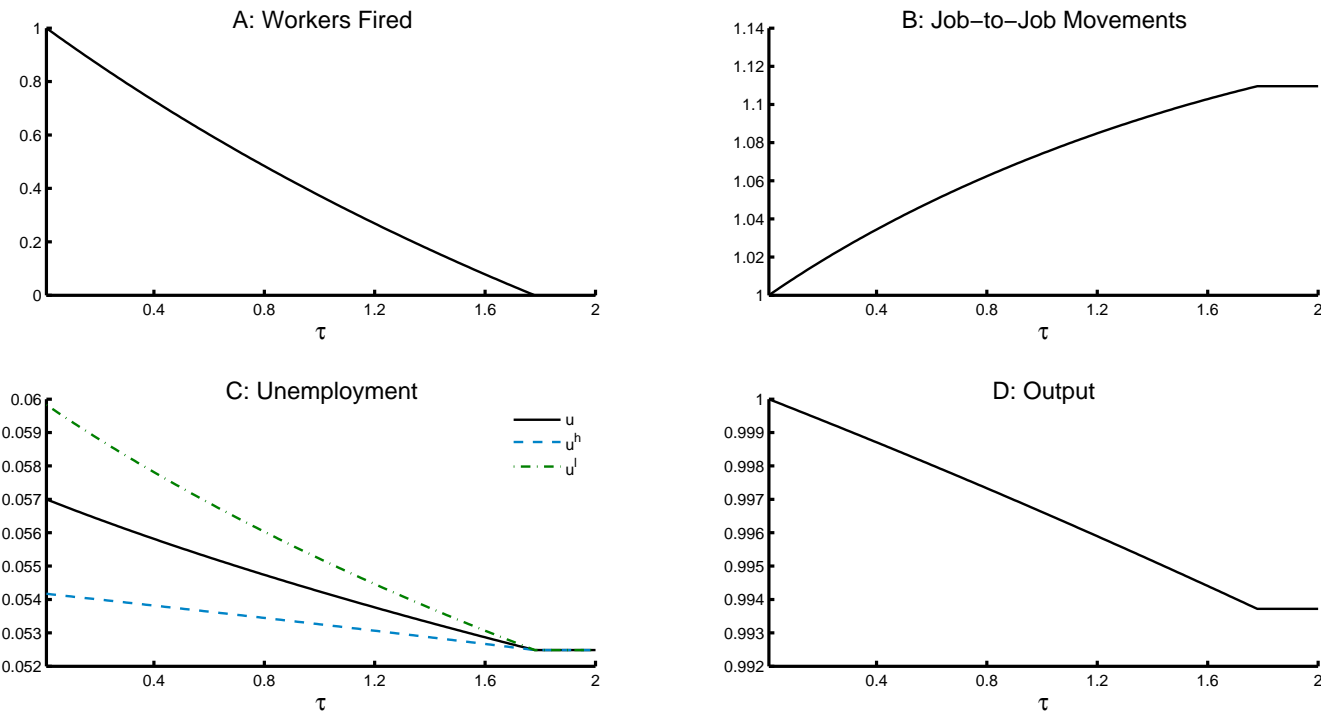
As in Mortensen and Pissarides (1999), unemployment drops in my model, and it drops for both types of workers (see Panel C of Fig. 1.5). This follows from two reasons: (i) low skill workers are less likely fired from good jobs; and (ii) bad firms benefit from higher firing costs, and thus, open more vacancies. But lower unemployment does not mean that resources are used more efficiently. Also consistent with Mortensen and Pissarides (1999), Panel D of Fig. 1.5 shows that output drops with the increase in firing costs, suggesting that the higher employment is applied in a less productive manner. In the model of Mortensen and Pissarides, output drops because firms require stronger negative productivity shocks to fire a worker. In my model, output also drops because good firms are forced to retain less productive workers. Moreover, the increase in the number of bad vacancies leads to lower unemployment but also lower average job productivity. Thus, it is misleading to look only at the number of jobs; it is also important to look at the quality of jobs.

The harmful effect of firing costs on output can be much larger in the context of economic growth. Although the model is not suited for the analysis of economic growth, I believe the reader would agree that a higher number of good jobs and a higher proportion of good jobs filled by high skill workers should foster growth. If that is the case, the static results in my model would extend to the rate of economic growth, with much larger negative effects on output.<sup>18</sup>

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<sup>18</sup>This is consistent with Bartelsman et al. (2011), Bassanini et al. (2009), and OECD (2010), among others, who focus on the harmful effect of firing costs on workers' reallocation with negative implications for economic growth.

Figure 1.5: The effect of firing costs on worker flows, unemployment, and output.



*Note:* The figure shows worker flows (workers fired in Panel A and workers who move from bad to good jobs in Panel B), unemployment, and output as a function of firing costs,  $\tau$ . In Panels A, B, and D, Worker flows and output are relative to the steady-state without firing costs ( $\tau = 0$ ). In Panel C, the figure shows the proportion of workers unemployed. In particular, it shows aggregate unemployment ( $u$ ), high skill unemployment ( $u^h$ ), and low skill unemployment ( $u^l$ ).



## 1.4.2 Sensitivity Analysis

Fig. 1.6 and Table 3.2 report the sensitivity analysis of the model. Fig. 1.6 shows how the proportion of workers fired changes the effect of firing costs. The parameter of interest is the highest productivity draw possible,  $\bar{a} = a^h + \underline{a}$ . A higher  $\bar{a}$  implies a smaller proportion of low skill workers fired because it is more likely to attain a productivity above the cutoff,  $a^* = 1.05$ .<sup>19</sup> Under the benchmark calibration ( $\bar{a} = 1.25$ ) the proportion of workers fired is 20%, but in Figure 1.6, this proportion moves from 33% ( $\bar{a} = 1.15$ ) to 14% ( $\bar{a} = 1.35$ ).

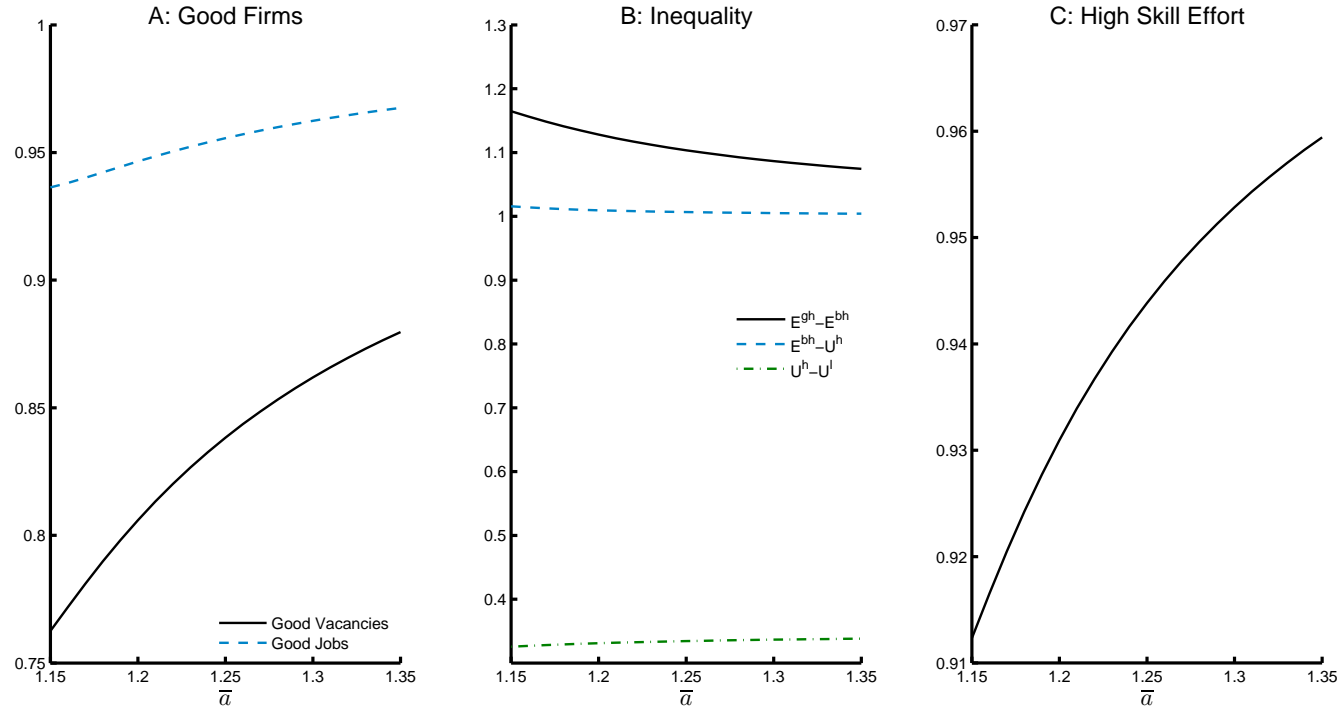
Clearly, the proportion of low skill workers fired has important quantitative implications: The effect of firing costs decreases as the proportion of workers fired drops. Yet, it does not have important qualitative implications: An increase in firing costs always deteriorates the proportion of good vacancies, strengthens the role of luck, and weakens the role of merit.

In Table 3.2, I also conduct a sensitivity analysis to the cutoff,  $a^*$ . A change in the initial (without firing costs) cutoff also changes the proportion of workers fired by good firms: A higher  $a^*$  implies a higher proportion of workers fired by good firms. These results corroborate the results in Figure 1.6. A higher proportion of workers fired (in the case of no firing costs) increases the effect of changes in the firing costs, but has no effect on the qualitative results of the model.

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<sup>19</sup>Remember that this proportion is  $\frac{a^* - \underline{a}}{\bar{a} - a^*}$ .

Figure 1.6: How does the proportion of low skill workers fired in good firms changes the effects of firing costs?



*Note:* The figure shows the ratio of the value in the steady-state with  $\tau = w^g$  with respect to the value in the steady-state without firing costs ( $\tau = 0$ ) as a function of the worker's maximum productivity draw,  $\bar{a}$ . It also shows how the proportion of low skill workers fired in good firms (by means of changes in the worker's productivity difference,  $\bar{a} - \underline{a}$ , for given cutoff,  $a^*$ ) changes the effects of firing costs: An increase in  $\bar{a}$  decreases the proportion of low skill workers fired in good firms. For example, for  $\bar{a} = 1.2$  the proportion is 25%, while for  $\bar{a} = 1.25$  the proportion is 20%.

Table 1.2: Sensitivity Analysis

	$z$			$E^{gh} - E^{bh}$			$E^{bh} - U^h$			$U^h - U^l$			$e^h$		
$\tau = 0$	$\tau = 0.5$	$\tau = 1$	$\tau = 2$	$\tau = 0.5$	$\tau = 1$	$\tau = 2$	$\tau = 0.5$	$\tau = 1$	$\tau = 2$	$\tau = 0.5$	$\tau = 1$	$\tau = 2$	$\tau = 0.5$	$\tau = 1$	$\tau = 2$
Benchmark	-8	-15	-24	5	9	16	0	1	1	-32	-60	-100	-3	-5	-9
$\alpha^* = 1.025$	-8	-13	-13	5	8	8	0	0	0	-59	-100	-100	-3	-4	-4
$\alpha^* = 1.075$	-8	-14	-26	5	9	18	1	2	4	-22	-43	-78	-3	-5	-10
$z = 0.30$	-3	-7	-13	1	3	6	0	0	0	-31	-60	-100	-1	-3	-6
$z = 0.70$	-14	-23	-34	10	18	29	1	2	3	-32	-60	-100	-4	-7	-11
$e^h = 0.15$	-5	-10	-18	2	5	9	1	2	4	-31	-60	-100	-2	-5	-8
$e^h = 0.50$	-13	-23	-35	10	20	33	-1	-1	-2	-32	-61	-100	-3	-5	-9
$\eta = 0.3$	-14	-23	-35	8	15	24	0	-1	-1	-33	-62	-100	-4	-8	-14
$\eta = 0.7$	-6	-11	-18	4	7	13	1	1	2	-31	-60	-100	-2	-4	-7
$y^g = 1.02$	-8	-15	-24	5	10	16	0	0	-1	-33	-63	-100	-3	-5	-9
$y^g = 1.10$	-7	-14	-24	5	9	16	2	3	5	-29	-57	-100	-2	-5	-9
$\alpha = 1.41$	-4	-8	-14	3	5	8	0	1	1	-31	-60	-100	-3	-6	-11
$\alpha = 2.30$	-10	-18	-29	6	12	20	1	1	1	-32	-61	-100	-3	-5	-9

*Note:* The table shows the robustness of the results of the model: It shows the percentage change when firing costs,  $\tau$ , increases from 0 to 0.5, 1, and 2, under different parametrizations of the model.

These results also give a tentative explanation for the conclusions of Bassanini et al. (2009) and OECD (2010). These studies conclude that economic growth falls by more in industries where employment protection more likely binds –so, in industries with a higher proportion of workers fired and low  $\bar{a}$  or high  $a^*$ . They suggest that it follows from lower workers’ reallocation. Besides suggesting workers’ reallocation, my results also suggest it can follow from a deterioration of the average quality of jobs in the economy. The intuition is simple. When  $\bar{a}$  is low (or  $a^*$  is high), good firms need to fire a higher a proportion of their workers. This implies that for a given change in firing costs, good firms must retain a higher proportion of workers who produce less than their wage. Thus, the value of opening a good firm decreases more rapidly with firing costs when  $\bar{a}$  is low or the cutoff (in the case of no firing costs) is high.

I also summarize other robustness checks in Table 3.2. I check whether the results of the model are robust to different elasticity of the matching function with respect to search effort,  $\eta$ , productivity of good firms,  $y^g$ , elasticity of the search on-the-job cost function with respect to effort,  $\alpha$ , initial proportion of good vacancies,  $z$ , and high skill workers initial search on-the-job effort,  $e^h$ . With some minor exceptions, the results do not change qualitatively: An increase in firing costs deteriorates the average quality of jobs, strengthens the role of luck, and weakens the role of merit. Quantitatively,  $y^g$  is almost irrelevant for the effects of firing costs. But the effects of firing costs are larger when  $\alpha$ ,  $z$ , and  $e^h$  are high, and when  $\eta$  is low.

## 1.5 Concluding remarks

In this chapter, I build a search and matching model in which high employment protection tends to generate a luck rewarding economy: When employment protection increases, the model predicts that luck tends to play a primary role in generating income inequality, while merit tends to play a minor –if not inexistent–role.

An increase in employment protection has major implications for incentives. I show that an increase in employment protection decreases incentives to search on-the-job, which amplifies the negative effect of firing costs on the value of good vacancies. Furthermore, there are other incentives that are not considered in the model. For example, agents invest in their education and develop their skills because they believe it will increase their future income. But if it only matters to be lucky, why should they try to be skilled?

## **Chapter 2**

# **Sticky-Price Models, Durable Goods, and Real Wage Rigidities**

### **2.1 Introduction**

Using the standard two sector New Keynesian model, Barsky et al. (2007) eloquently demonstrate that the degree of price flexibility in the durable goods sector dictates the response of aggregate output to a monetary shock. When nondurable-goods' prices are sticky but durable-goods' prices are flexible, the outputs of the two sectors move in opposite directions, leaving aggregate output unchanged. Aggregate output, however, reacts significantly when nondurable- and durable-goods' prices are sticky. But then nondurables output virtually does not react. In other terms, the standard New Keynesian model generates either negative sectoral comovement and aggregate neutrality or no sectoral comovement.

Yet, VAR evidence overwhelmingly suggests positive comovement in the aftermath of a monetary shock (see, for example, Barsky et al. (2003), Erceg and Levin 2006, Monacelli 2009). Two stylized facts are that (i) aggregate output and sectoral outputs move together, and (ii) price and output of the durable sector react more strongly than those of the nondurable sector. Moreover, the distinctive feature of a business cycle is that the output of many sectors of the economy move together. Therefore, the New Keynesian model, the workhorse in analysis

of monetary business cycles, needs to be reconciled with these facts. We do so simply by introducing real wage rigidities into an otherwise standard two sector New Keynesian model.

In Section 2, we document VAR evidence for the U.S. Economy. Our evidence confirms the results of previous studies: The outputs of the durable and nondurable sector comove. It also confirms the results of Altig et al. (2011), Amato and Laubach (2003), Christiano et al. (2005): Our evidence confirms that the reaction of real wages to monetary shocks is rather muted. One other supporting argument for the existence of real wage rigidities flows from the Dunlop-Tarshis observation –that hours worked and real wages are uncorrelated (Dunlop 1938 and Tarshis 1939). As a modeling feature, the crucial role of real wage rigidities in propagating shocks has long been recognized as well. In his review of the business cycle theory, Lucas (1981) argues that models relying on systematic real wage movements are *doomed to failure*. Ball and Romer (1990) show that real wage rigidities amplify the effects of nominal rigidities. Hall (2005b) shows that real wage rigidities generate volatile unemployment and vacancies in the context of search and matching models. Blanchard and Galí (2007) show that real wage rigidities generate a trade-off between output and inflation stabilization. More recently, Shimer (2012b) shows that real wage rigidities can account for jobless recoveries.

To model real wage rigidities, we follow Blanchard and Galí (2007): We modify the labor supply equation of the standard New Keynesian model by assuming that real wages are a weighted sum of lagged real wages and the marginal rate of substitution between consumption and leisure. Without any other change to the standard model, this simple modification removes the above mentioned puzzling theoretical results, i.e., it obtains non-neutral money and positive comovement between sectoral outputs. Using our modification we first show that when durable-goods' prices are *perfectly flexible*, (i) durables output reacts sharply to monetary shocks; (ii) aggregate output and nondurables output react similarly and less dramatically than durables output; (iii) the durable-price index reacts more strongly than the nondurable-price index. Using our modification we then show that these

three results also hold as long as durable-goods' prices are (slightly) *more flexible* than nondurable-goods' prices.<sup>1</sup>

The economic intuition behind our results is also simple: Real wage rigidities decrease the elasticity of marginal costs, suppressing the reaction of prices in both sectors. This, in turn, renders positive comovement and aggregate non-neutrality possible.

Our reconciliation of the two sector New Keynesian model is not the first one. Bouakez et al. (2011) and Sudo (2012) argue that inputoutput interactions help solve the comovement problem. In the models employed in both papers, because the production of nondurable goods requires durable goods as inputs, and vice versa, and because nondurable-goods' prices are sticky, the pass-through of the monetary impulse into durable-goods' prices is limited. Accordingly, durables output can move in the same direction of the change in aggregate demand. Carlstrom and Fuerst (2010) document that adding three features –sticky nominal wages, adjustment costs in housing construction, and habit formation in consumption– into an otherwise standard model brings it closer to reality. Sticky nominal wages is enough to generate sectoral comovement in the first quarter. But adjustment costs in housing construction are required to generate sectoral comovement for more than one quarter. Monacelli (2009) and Sterk (2010) discuss the role of credit market frictions in accounting for positive sectoral comovement.

We think that our approach has the advantages of parsimony and robustness. The way we modify the standard model is very simple: We replace only one equation to obtain our model with real wage rigidities. Our results are robust to all reasonable parameter values.

In the remainder of the chapter, we start by documenting VAR evidence for the U.S. economy in Section 2.2. To make our chapter self-contained and our

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<sup>1</sup>The condition that durable-goods' prices are more flexible than nondurable-goods' prices is in line with the empirical evidence documented in the literature on micro-price data (for a survey, see Klenow and Malin 2010).



model easy to compare to the one analyzed in Barsky et al. (2007), in Section 2.3 we briefly present their benchmark model, which, hereafter, we call the standard model. In Section 2.4, we incorporate real wage rigidities simply by modifying the labor supply equation of the standard model. Also in the same section we analytically assess the role of real wage rigidities in the neutrality and comovement problems. In Section 2.5, we calibrate the two models, and present the results of our numerical simulations. In Section 2.6, we offer some concluding remarks.

## 2.2 Estimation

In this section, we empirically assess the behavior of durable goods consumption, nondurable goods consumption, and real wages in response to identified monetary shocks in the U.S. economy. We do so by means of a quarterly VAR model for the US economy that we set out as follows:

$$y_t = \sum_{j=1}^L A_j y_{t-j} + B\epsilon_t. \quad (2.1)$$

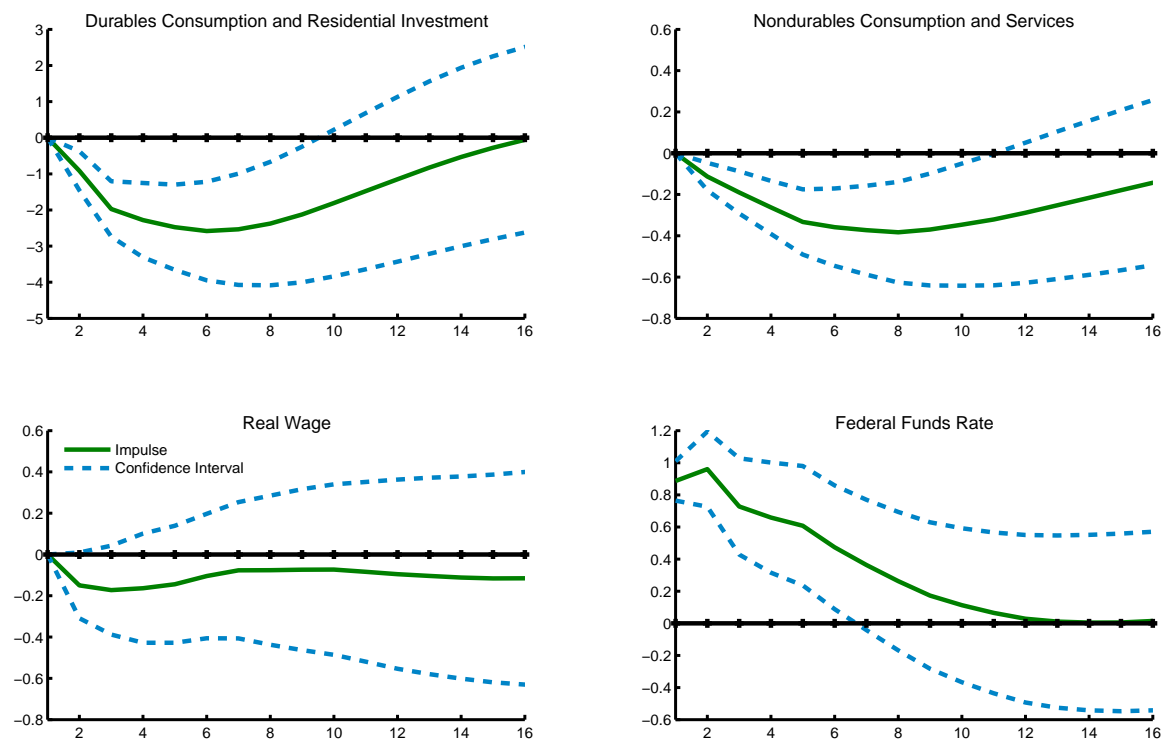
$y_t$  is a vector of four variables: Real durable consumption, real nondurable consumption, real wages, and the federal funds rate; and  $\epsilon_t$  is a vector of contemporaneous disturbances.<sup>2</sup> All variables, except for the federal funds rate, are in logs. Furthermore, we estimate the VAR with a constant, a linear trend, four lags, and for the period 1980:1 to 2007:4.<sup>3</sup>

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<sup>2</sup>We define: (i) real durables consumption as the Törnqvist index of the sum of personal consumption expenditure in durable goods with investment housing, both obtained from the Bureau of Economic Analysis; (ii) real nondurable consumption as the Törnqvist index of the sum of personal consumption expenditures in nondurables goods with services, both also obtained from the Bureau of Economic Analysis; and (iii) real wages as real compensation per hour in the nonfarm business sector, obtained from Federal Reserve Economic Data. The federal funds rate is also from the Federal Reserve Economic Data.

<sup>3</sup>As argued by Amato and Laubach (2003), we estimate the VAR over a period that policy is most likely characterized by constant parameters: The great moderation period. Therefore, our sample starts in 1980:1 as Amato and Laubach (2003) and finishes in 2007:4 before the global burst of the financial crisis.

Figure 2.1: Estimated Impulse Responses



*Note:* The figure shows estimated impulse responses to a monetary policy tightening (Sample period 1980:1–2007:4; 95% confidence bands).

Our estimation results confirms the results of previous studies: (i) Monetary policy has a statistically insignificant effect in real wages, which confirms the results of Altig et al. (2011), Amato and Laubach (2003), and Christiano et al. (2005); (ii) durable consumption comoves with nondurable consumption, which confirms the results of Barsky et al. (2003); (iii) and durable consumption has a much higher peak response than nondurable consumption, which also confirms Barsky et al. (2003). Our assessment, however, has the advantage to confirm these stylized facts in a single VAR estimation.

## 2.3 The Model

### 2.3.1 The Household

The household supplies labor,  $n_t$ , and capital,  $k$ , to the firms in durable and non-durable sectors. The stock of capital is fixed. Because production factors are perfectly mobile, the prices of these factors do not differ across sectors. The household chooses the consumption of nondurable goods,  $c_t$ , the stock of durable goods,  $d_t$ , labor supply, and purchases of durable goods,  $x_t$ , to maximize her utility

$$E_t \left[ \sum_{i=0}^{\infty} \beta \left( \psi_c \ln c_{t+i} + \psi_d \ln d_{t+i} - \frac{\phi}{2} n_{t+i}^2 \right) \right],$$

subject to the budget constraint

$$\frac{p_{c,t}c_t}{p_t} + \frac{p_{x,t}x_t}{p_t} + \frac{m_t}{p_t} \leq w_t n_t + \Pi_t + t_t + \frac{m_{t-1}}{p_t} + r_t k,$$

and the law of motion for the stock of durable goods

$$d_t = x_t + (1 - \delta)d_{t-1}, \quad (2.2)$$

where  $w_t$ ,  $\Pi_t$ ,  $t_t$ , and  $r_t$  are the real wage, the real dividend income from owning intermediate firms, real lump-sum transfers, and the real rental price of capital, respectively;  $m_t$  is nominal money balances;  $p_t$  is the GDP deflator; and  $p_{c,t}$  ( $p_{x,t}$ ) is

the price index of the composite nondurable (durable) good. Regarding the parameters,  $\delta > 0$  is the rate of depreciation of the stock of durable goods,  $0 < \beta < 1$  is the discount factor,  $\psi_c > 0$  and  $\psi_d > 0$  are the weights of nondurable and durable goods in the subutility, and  $\phi > 0$  measures the disutility from labor.<sup>4</sup>

Let  $\lambda_t$  and  $\mu_t$  be the Lagrange multipliers associated with the constraints above. The first order conditions to the household's problem are then

$$\psi_c c_t^{-1} = \frac{\lambda_t p_{c,t}}{p_t}, \quad (2.3)$$

$$\mu_t = \psi_d d_t^{-1} + \beta(1 - \delta)E_t[\mu_{t+1}], \quad (2.4)$$

$$\phi n_t = \lambda_t w_t, \quad (2.5)$$

$$\frac{\lambda_t p_{x,t}}{p_t} = \mu_t. \quad (2.6)$$

### 2.3.2 Firms

Within both sectors, there are perfectly competitive final good producers and monopolistically competitive intermediate good producers. Because the structure of production is symmetric across sectors, below we use a generic letter,  $j = c, x$ , to denote any of the sectors.

The production technology for the final good  $j_t$  is given by

$$j_t = \left[ \int_0^1 j_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \quad (2.7)$$

where  $j_t(i)$  is a differentiated intermediate good. The elasticity of substitution among intermediate goods,  $\epsilon > 1$ , is assumed to be the same in both sectors. Denoting the price of good  $i$  in sector  $j$  by  $p_{j,t}(i)$ , profit maximization of final

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<sup>4</sup>The felicity  $\psi_c \ln c_t + \psi_d \ln d_t - \frac{\phi}{2} n_t^2$  implies that the intertemporal elasticity of substitution, the intratemporal elasticity of substitution between durable and nondurable consumption, and the Frisch labor supply elasticity equal one.

good producers implies the demand function

$$j_t(i) = \left( \frac{p_{j,t}(i)}{p_{j,t}} \right)^{-\epsilon} j_t. \quad (2.8)$$

Together with eq. 2.8, zero profits of final goods producers imply the sectoral price index

$$p_{j,t} = \left[ \int_0^1 p_{j,t}(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}. \quad (2.9)$$

The production technology for an intermediate good is given by

$$j_t(i) = k_{j,t}(i)^\alpha n_{j,t}(i)^{1-\alpha}, \quad (2.10)$$

where  $k_{j,t}(i)$  and  $n_{j,t}(i)$  are the capital and labor services hired by firm  $i$  operating in sector  $j$ . Prices are set a la Calvo (1983). Specifically, each period, only a  $1 - \theta_j$  fraction of intermediate good producers can reset their prices. Then profit maximization implies the demands for labor and capital services

$$w_t = (1 - \alpha) \frac{j_t(i)}{n_{j,t}(i)} mc_t, \quad (2.11)$$

$$r_t = \alpha \frac{j_t(i)}{k_{j,t}(i)} mc_t, \quad (2.12)$$

and the optimal price of good  $i$

$$p_{j,t}^*(i) = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{s=0}^{\infty} (\theta_j \beta)^s E_t[\lambda_{t+s} p_{j,t+s}^\epsilon j_{t+s} mc_{t+s}]}{\sum_{s=0}^{\infty} (\theta_j \beta)^s E_t[\lambda_{t+s} p_{j,t+s}^\epsilon j_{t+s} p_{t+s}^{-1}]}, \quad (2.13)$$

where  $mc_t$  is the real marginal cost. Note that the real marginal cost is the same in both sectors. This stems from the Cobb-Douglas production function and the perfect mobility of production factors.

Because of symmetry across intermediate firms within each sector, intermediate firms who can reset their prices set the same price. And because only a  $1 - \theta_j$

fraction of prices can be reset every period, and, thus, the remaining fraction remains unchanged, the sectoral price index, eq. 2.9, is now read

$$p_{j,t}^{1-\epsilon} = \theta_j p_{j,t-1}^{1-\epsilon} + (1 - \theta_j) p_{j,t}^{*1-\epsilon}. \quad (2.14)$$

### 2.3.3 Aggregation, Real GDP, and Money

At any point in time, production factors can be divided between the durable and nondurable sectors according to

$$n_t = n_{c,t} + n_{x,t}, \quad (2.15)$$

$$k = k_{c,t} + k_{x,t}, \quad (2.16)$$

where  $n_{c,t}$  and  $k_{c,t}$  ( $n_{x,t}$  and  $k_{x,t}$ ) are labor and capital hired in the nondurable (durable) sector. Market clearing implies  $n_{j,t} = \int_0^1 n_{j,t}(i) di$  and  $k_{j,t} = \int_0^1 k_{j,t}(i) di$  for  $j = c, x$ .

Real GDP,  $y_t$ , is defined by using the steady state values of the sectoral price indices in nominal GDP

$$y_t \equiv \bar{p}_c c_t + \bar{p}_x x_t. \quad (2.17)$$

Hence the GDP deflator is obtained by

$$p_t = \frac{p_{c,t} c_t + p_{x,t} x_t}{y_t}. \quad (2.18)$$

The demand for money is motivated simply by assuming that it is proportional to nominal GDP

$$m_t = p_t y_t. \quad (2.19)$$

Any difference in money supply from one period to the next is distributed to the household through lump-sum transfers  $p_t t_t = m_t - m_{t-1}$ . And the (log) growth rate of money supply is simply a mean zero i.i.d. random variable:

$$\ln \frac{m_t}{m_{t-1}} = \varepsilon_t. \quad (2.20)$$

## 2.4 Real Wage Rigidities

In our model with real wage rigidities, the real wage differs from the marginal rate of substitution between consumption and leisure – i.e., eq. 2.5 does not hold. To model real wage rigidities, we follow Blanchard and Galí (2007) ad hoc but parsimonious formulation. Namely, we assume that the (log) real wage for which the household members are willing to work is a weighted sum of the lagged (log) real wage and the marginal rate of substitution between consumption and leisure

$$\ln w_t = \gamma \ln w_{t-1} + (1 - \gamma) \ln mrs_t, \quad (2.21)$$

where  $0 \leq \gamma \leq 1$  measures the degree of real wage rigidities in the economy, and  $mrs_t$  is the marginal rate of substitution between consumption and leisure,  $mrs_t \equiv \frac{\phi n_t}{\lambda_t} = \frac{p_{x,t} \phi n_t}{p_t \mu_t}$ . That is, to obtain our model, we replace eq. 2.5 in the standard model with eq. 2.21.

Next we analytically assess the role of real wage rigidities in the comovement problem arising under flexibly priced durable goods. Following the neat analysis of Barsky et al. (2007) we display a crucial property of the shadow value of durable goods:  $\mu_t$  is nearly invariant. To this end, we rewrite eq. 2.4 as

$$\mu_t = \psi_d E_t \left[ \sum_{i=0}^{\infty} (\beta(1 - \delta))^i d_{t+i}^{-1} \right].$$

Two remarks are in order about the shadow value of durable goods. First, the last equation states that the shadow value of durable goods is the expected sum of the discounted value of marginal utilities of durable goods. With low values of  $\delta$ , temporary changes in the marginal utility of durable goods have insignificant effects on their shadow value. Second, because the stock-flow ratio of durable goods is high ( $d/x = 1/\delta$  in the steady state), the effect of purchases of durable goods on the stock is also insignificant. These two properties justify treating the shadow value of durable goods as constant in our analytical exposition (i.e.,  $\mu_t \approx \mu$ ).

To continue with our analysis of the role of real wage rigidities, we first rewrite

eq. 2.21 as  $\left(\frac{w_t}{w_{t-1}}\right)^{\frac{\gamma}{1-\gamma}} = mrs_t = \frac{p_{x,t}\phi n_t}{p_t\mu_t}$ . To substitute  $p_{x,t}/p_t$  out from the last expression, recall that durable-goods' prices are flexible, and that the capital-labor ratio is common to all firms, implying  $x \frac{p_{x,t}}{p_t} = \frac{\epsilon}{\epsilon-1} mc_t = \frac{\epsilon}{\epsilon-1} \frac{w_t}{(1-\alpha)} \left(\frac{n_t}{k}\right)^\alpha$ . This together with  $\mu_t \approx \mu$  enable us to rewrite eq. 2.21 as

$$\left(\frac{w_t}{w_{t-1}}\right)^{\frac{\gamma}{1-\gamma}} \approx \frac{\epsilon\phi k^{-\alpha}}{(\epsilon-1)(1-\alpha)\mu} n_t^{1+\alpha}. \quad (2.22)$$

It is now easy to see that in the standard model, i.e.,  $\gamma = 0$ , the only solution to eq. 2.22 is invariant aggregate employment. Also, because  $n_t = n_{c,t} + n_{x,t}$ , positive comovement between sectoral employment levels is impossible. But if  $\gamma > 0$ , aggregate employment moves in the direction of the change in the real wage, and sectoral employment levels may move together. The economic intuition is straightforward. Once the real wage is rigid, marginal costs, common to all sectors, become less sensitive to changes in aggregate demand. This, in turn, limits the extent to which prices in both sectors react to changes in aggregate demand, rendering it possible for the output in both sectors to move together.

To confirm these results and to provide further details, next we report the results of our numerical simulations, using the log-linear approximation to the model around the nonstochastic steady state.

## 2.5 Numerical Simulations

### 2.5.1 Calibration

We start by calibrating the parameters. Our choice of parameter values is the same made by Barsky et al. (2007), except that we calibrate our model to quarterly data. Table 3.1 summarizes our choice of parameters targeting the following steady state values. Namely we set an annual discount rate of 2% implying  $\beta = 0.9951$ . We set an annual depreciation rate of 5% implying  $\delta = 0.0123$ . We set the shares of sectoral outputs in GDP such that in the steady state the output of the nondurable



Table 2.1: Benchmark Parameters

Discount factor:	$\beta = 1.02^{-1/4}$
Rate of depreciation:	$\delta = 1.05^{1/4} - 1$
Relative weight in subutility:	$\psi_c/\psi_d = 3\delta/(1 - \beta(1 - \delta))$
Capital share:	$\alpha = 0.35$
Degree of price stickiness	
in the nondurable sector:	$\theta_c = 0.6534$
in the durable sector:	$\theta_x = 0$
Degree of real wage rigidities:	$\gamma = 0.9546$

sector is thrice that of the durable sector implying  $\psi_c/\psi_d = 2.1467$ . We set a capital share of 35% implying  $\alpha = 0.35$ .

As a benchmark, we set an half-life of two quarters for nominal stickiness in the nondurable sector implying  $\theta_c = 0.6534$ . The comovement and aggregate neutrality problems are immense when durable-goods' prices are flexible. For this reason we set  $\theta_x = 0$ . Yet, in Section 2.5.2 we also consider a wide range of values for  $\theta_c$  and  $\theta_x$ . The parameters  $\phi$  and  $\epsilon$  do not play any role in the log-linear model.

In all our experiments, we study the response of key macroeconomic aggregates to a permanent increase in money supply: The growth rate of money supply,  $\varepsilon_t$ , assumes 0.01 at  $t = 1$  and zero thereafter. Thus the monetary shock expands the money supply once-and-for-all by 1%.

We are not aware of a direct empirical evidence for the parameter governing the degree of real wage rigidities. In the literature, the values assumed range from 0.5 to 1. Blanchard and Galí (2007) set  $\gamma = 0.9$  in their baseline calibration and also experiment both with  $\gamma = 0.8$  and  $\gamma = 0.5$ . Duval and Vogel (2007) experiment with  $\gamma = 0.79$  and  $\gamma = 0.93$ . Shimer (2012a) assumes that the real wage rate is constant in business cycle frequencies, implying  $\gamma = 1$ . To set a benchmark value for  $\gamma$ , we target the standard deviation of US GDP in a version of our model

that also allows for total factor productivity shocks. To produce a standard deviation of GDP of 1.72%, we choose  $\gamma = 0.9546$ .<sup>5</sup> Because this parameter is key to our discussion, we start by assuming not a single value but a range:  $\gamma \in [0, 1]$ .

## 2.5.2 Results

### The Role of $\gamma$

First we study how the degree of real wage rigidities,  $\gamma$ , affects the responses of four selected variables –GDP, the real wage, durables output, nondurables output– to a permanent 1% increase in money supply.<sup>6</sup> To this end, we compute the first quarter and the first year (quarterly averaged) responses of these four variables for  $\gamma \in [0, 1]$  with a grid of 0.01. Figure 2.2 illustrates the two measures as a function of  $\gamma$  (first quarter as solid line; first year as dashed line).

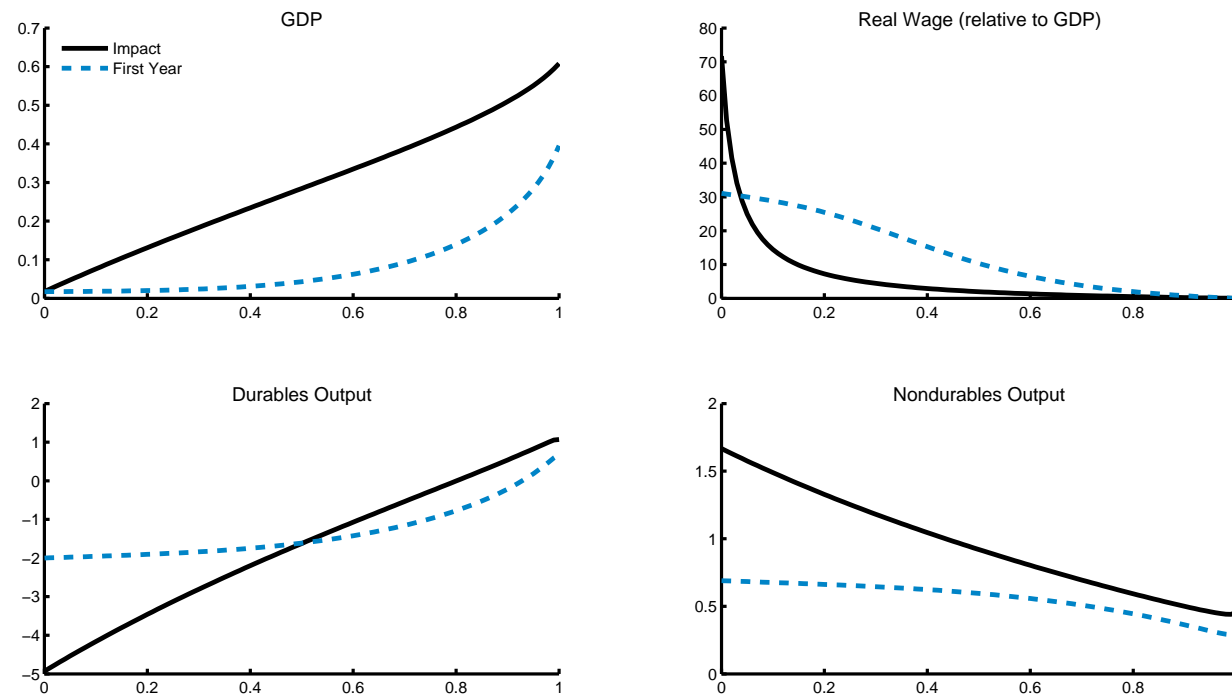
Eyeballing the graphs related to GDP and the two sectors' outputs, we observe that as  $\gamma$  increases, both problems, aggregate neutrality and negative sectoral comovement, cease to exist. Simultaneously, the first quarter and the first year responses of the relative real wage approach empirically plausible values.

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<sup>5</sup>Specifically, we assume that the production technology for an intermediate good is given by  $j_t(i) = a_t k_{j,t}(i)^\alpha n_{j,t}(i)^{1-\alpha}$ , where  $a_t$  is total factor productivity and follows  $\log(a_t) = 0.95 \log(a_{t-1}) + \nu_{a,t}$ . The growth rate of money supply,  $\varepsilon_t$ , follows  $\log(\varepsilon_t) = 0.49 \log(\varepsilon_{t-1}) + \nu_{m,t}$ . The innovations  $\nu_{m,t}$  and  $\nu_{a,t}$  are mean zero i.i.d. random variables with standard deviations of 0.89% and 0.7%. The correlation between the innovations is set to zero. The smoothing parameter of the Hodrick-Prescott filter is set to 1600. Accordingly, 85% of the deviation in output are due to shocks to total factor productivity.

<sup>6</sup>For the sake of brevity, we select only four variables. We select GDP to address aggregate neutrality; we select the sectoral outputs to address the comovement problem; and we select the relative real wage to address real wage rigidities.

Figure 2.2: The Role of Real Wage Rigidities



*Note:* The horizontal axis measures the degree of real wage rigidities,  $\gamma \in [0, 1]$ . The vertical axis measures the impact (solid line) and first year quarterly average (dashed line) responses to a permanent monetary expansion. The rest of the parameters assume their benchmark values.

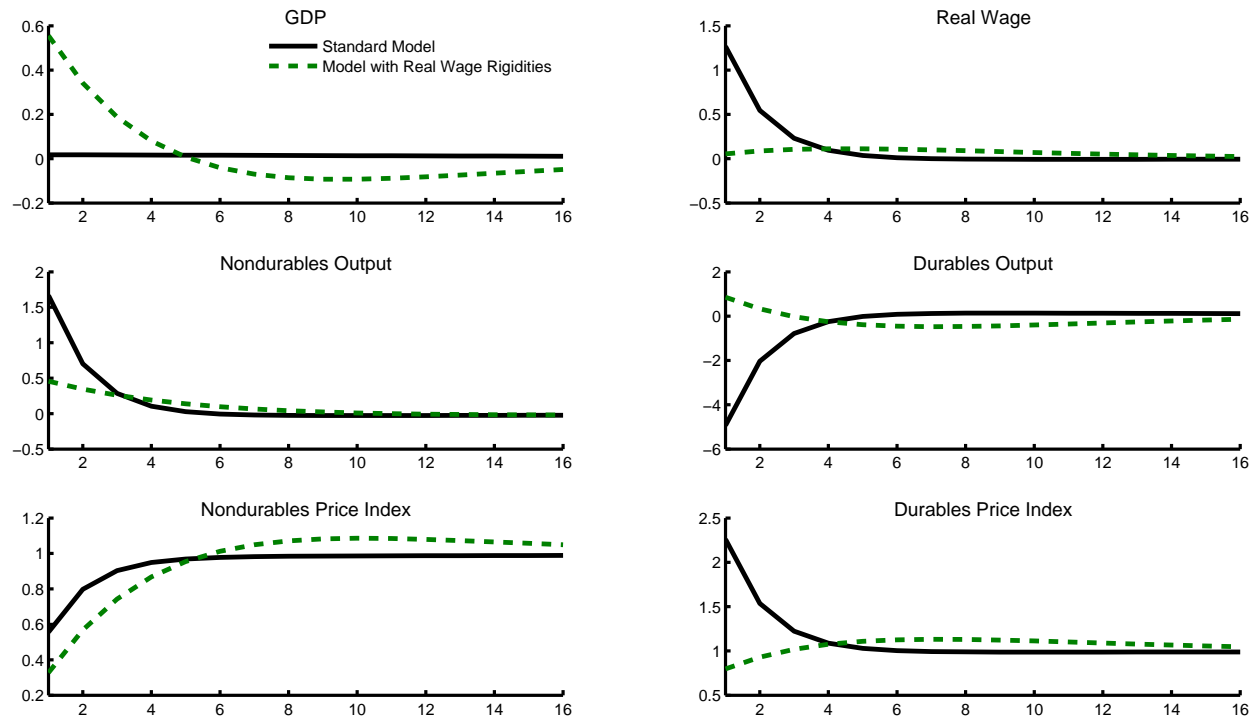
The response of GDP is always positive and increasing in  $\gamma$ . More specifically, the first year response starts increasing at a faster rate around  $\gamma = 0.8$ ; for  $\gamma \in [0.8, 1]$  aggregate non-neutrality is significant. The interval  $\gamma \in [0.8, 1]$  also suffices to generate positive comovement between the sectoral outputs at first quarter responses. Regarding first year responses, the threshold value of  $\gamma$  generating positive comovement between sectoral outputs is 0.93. In the standard model, i.e., when  $\gamma = 0$ , the first quarter response of the real wage is 70 times higher than that of aggregate output. By contrast, the response of the relative real wage decreases in  $\gamma$  as implied by the law of motion of the real wage, eq. 2.21. With our baseline calibration,  $\gamma = 0.9546$ , the first quarter response of the real wage is 10% of that of aggregate output.

### **Impulse Response Functions**

To gain further intuition, in Figure 3.2 we contrast the standard model ( $\gamma = 0$ ; solid line) with the model with real wage rigidities ( $\gamma = 0.9546$ ; dashed line) in terms of impulse response functions. Again, the innovation is a permanent 1% increase in money supply, and durable-goods' prices are flexible.

In both models, the impulse response functions for the durable sector illustrate that the reaction of the output is the mirror image of that of the price index. The contrast is primarily on impact effects. In the model with real wage rigidities, the durable-price index, on impact, undershoots its long-run value by 0.2%. In the standard model, however, it overshoots by 1.5%. Because of these temporary changes in prices, in the standard model durables output decreases on impact by 5%, while with real wage rigidities it increases by 0.85%. Thus the most striking aspect of incorporating real wage rigidities into the standard model is the reversal of the durable-output response: The household switches expenditure from non-durable to durable goods. This switch in expenditure also attenuates the impact response of nondurables output and price index.

Figure 2.3: Impulse Response Functions



*Note:* The horizontal axis measures time in quarters. The vertical axis measures the logarithmic deviation from the steady state. The impulse is a permanent monetary expansion. Solid lines represent the responses in the standard model,  $\gamma = 0$ , while the dashed lines represent the responses in the model with real wage rigidities,  $\gamma = 0.9546$ . The rest of the parameters assume their benchmark values.

Obviously, the key variable to understand the under- and overshooting of the durable-price indices, and, thus, the reversal of the durable-output response, is the reaction of the real wage. In the model with real wage rigidities, the response of the real wage is muted, being in line with the empirical evidence documented in VAR studies (see, for example, Altig et al. 2011, Amato and Laubach 2003, and Christiano et al. 2005). With real wage rigidities, thanks to this muted response of the real wage, the durable-price index can under-shoot its long-run value, allowing the durable-output to react positively on impact to monetary expansions.

In the standard model, because the sectoral outputs move in opposite directions, GDP, which is the weighted average of the sectoral outputs, is essentially unchanged. In the model with real wage rigidities, the responses of the sectoral outputs increase simultaneously, and, thus, the response of GDP increases as well. In short, with real wage rigidities, we obtain non-neutral money and positive co-movement between sectoral outputs.

Ordering the magnitudes of peak-responses of the sectoral variables and GDP reveals that in the model with real wage rigidities: (i) Durables output reacts sharply to monetary shocks; (ii) GDP and nondurables output react similarly and less dramatically than durables output; (iii) The durable-price index reacts more strongly than the nondurable-price index. These three results are also in line with the stylized facts documented in Barsky et al. (2003), Erceg and Levin (2006), and Monacelli (2009).

### **The Roles of $\theta_c$ and $\theta_x$**

As in the majority of the related literature, in our evaluation so far we have only considered the case of flexibly priced durable goods.<sup>7</sup> In their survey of the micro-price studies analyzing data that underlie U.S. consumer and producer price indices, Klenow and Malin (2010) document that while price flexibility increases

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<sup>7</sup>See Bouakez et al. (2011), Carlstrom and Fuerst (2010), and Sudo (2012). Bouakez et al. experiment also with an half life of one month for nominal stickiness in the durable sector.

with durability, durable-goods' prices, on average, are not perfectly flexible.

To detect how the imperfect flexibility of durable-goods' prices affects the nature of comovement in both models we evaluate, we first compute the first-year effects of a permanent increase in money supply on nondurables and durables outputs for all possible combinations of ten equally distanced values of  $\theta_c$  and  $\theta_x$  in the range  $[0.1, 0.9]$ . To produce a basic summary statistic of sectoral comovement, we, then, compute the ratio of the first year multiplier of nondurables output to that of durables output; positive and negative values implying positive and negative comovement, respectively. Focusing only on the cases in which durable-goods' prices are more flexible than the nondurable-goods' prices,  $\theta_x \leq \theta_c$ , Table 3.2 presents how our summary statistic changes with the sectoral price flexibility in the standard model (Panel A) and in the model with real wage rigidities (Panel B).<sup>8</sup> In Table 3.2, we use bold-faced type to highlight the cases in which the statistic falls into the empirically plausible range of  $[10\%, 50\%]$  (Barsky et al. 2003, Erceg and Levin 2006, and Monacelli (2009)).

Regarding the standard model, from Panel A of Table 3.2 we observe that in 30 of all 36 cases considered we obtain negative comovement. In two cases we obtain positive comovement but the effect on the nondurable-output exceeds that on the durable-output. Only in four cases our statistic lies within the empirically plausible range. Regarding the model with real wage rigidities, from Panel B of Table 3.2 we observe that in all cases we obtain positive comovement, and the response of the nondurables-output is less than that of the durable-output. In 21 cases, the statistic lies within the empirically plausible range.

Thus, Table 3.2 confirms our previous argument that in the model with real wage rigidities the comovement problem disappears as long as durable-goods' prices are more flexible than nondurable-goods' prices.

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<sup>8</sup>In the cases of equal flexibility,  $\theta_x = \theta_c$ , for the two models considered, the statistic ranges between 0.009 and 0.035 implying the lack of sectoral comovement. In the cases of extra flexibility in the nondurable-goods' prices,  $\theta_x > \theta_c$ , again for the two models considered, the statistic ranges between  $-0.020$  and  $-0.199$  implying negative comovement.

Table 2.2: Sectoral Comovement &amp; Price Stickiness

Panel A: Standard Model ( $\gamma = 0$ )								
	$\theta_c = 0.2$	$\theta_c = 0.3$	$\theta_c = 0.4$	$\theta_c = 0.5$	$\theta_c = 0.6$	$\theta_c = 0.7$	$\theta_c = 0.8$	$\theta_c = 0.9$
$\theta_x = 0.1$	-0.746	-0.458	-0.405	-0.383	-0.372	-0.365	-0.361	-0.363
$\theta_x = 0.2$		-6.730	-0.613	-0.468	-0.419	-0.396	-0.383	-0.380
$\theta_x = 0.3$			1.319	-0.833	-0.534	-0.454	-0.419	-0.405
$\theta_x = 0.4$				0.679	-1.183	-0.609	-0.492	-0.449
$\theta_x = 0.5$					<b>0.479</b>	-1.920	-0.701	-0.542
$\theta_x = 0.6$						<b>0.374</b>	-5.112	-0.850
$\theta_x = 0.7$							<b>0.305</b>	4.956
$\theta_x = 0.8$								<b>0.245</b>

Panel B: The Model with Real Wage Rigidities ( $\gamma = 0.9546$ )								
	$\theta_c = 0.2$	$\theta_c = 0.3$	$\theta_c = 0.4$	$\theta_c = 0.5$	$\theta_c = 0.6$	$\theta_c = 0.7$	$\theta_c = 0.8$	$\theta_c = 0.9$
$\theta_x = 0.1$	<b>0.252</b>	<b>0.496</b>	0.699	0.833	0.896	0.896	0.838	0.689
$\theta_x = 0.2$		<b>0.145</b>	<b>0.291</b>	<b>0.431</b>	0.548	0.626	0.650	0.581
$\theta_x = 0.3$			<b>0.110</b>	<b>0.220</b>	<b>0.331</b>	<b>0.427</b>	<b>0.490</b>	<b>0.477</b>
$\theta_x = 0.4$				0.093	<b>0.185</b>	<b>0.278</b>	<b>0.355</b>	<b>0.379</b>
$\theta_x = 0.5$					0.083	<b>0.164</b>	<b>0.242</b>	<b>0.287</b>
$\theta_x = 0.6$						0.078	<b>0.149</b>	<b>0.204</b>
$\theta_x = 0.7$							0.074	<b>0.132</b>
$\theta_x = 0.8$								0.071

*Note:* The table gives the ratio of first year multiplier of nondurables output to that of durables output. A negative value implies negative sectoral comovement. Bold-faced type is used to highlight the cases in which the ratio falls into the empirically plausible range: [10%, 50%].



## 2.6 Concluding Remarks

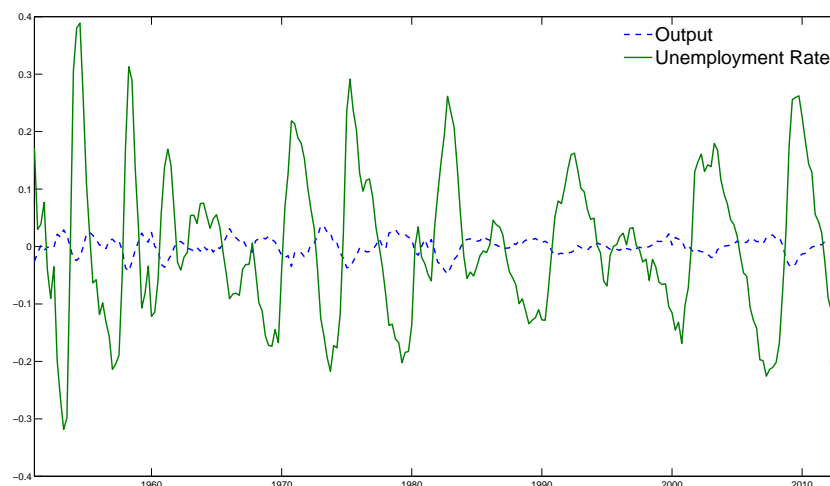
In this chapter, we argue that real wage rigidities are a missing element in the standard two sector New Keynesian model. In the standard model, for aggregate output to increase following a monetary shock, durable-goods' prices must be sticky. If durable-goods' prices are flexible, and nondurable-goods' prices are sticky, then the standard model predicts negative sectoral comovement besides aggregate neutrality. If both durable- and nondurable-goods' prices are sticky, then the standard model predicts no sectoral comovement. By incorporating real wage rigidities into the standard model, we obtain positive sectoral comovement and, thus, aggregate non-neutrality as long as durable-goods' prices are more flexible than nondurable-goods' prices.

Real wage rigidities have been advocated as a missing element also in other contexts (Blanchard and Galí 2007, Hall 2005b, and Shimer (2012a) 2012a, just to name a few recent examples). Our research thus justifies our agreement with Shimer (2012a) that wage rigidities should be put back into the center of research on macroeconomics. This chapter is a contribution to that program.

## Chapter 3

# A New Perspective on the Unemployment Volatility Puzzle

Figure 3.1: Unemployment and Output



*Note:* Quarterly data (1951:1–2012:4), in logs, filtered using the Hodrick-Prescott filter with a smoothing parameter of 1600.

### 3.1 Introduction

Figure 3.1 displays postwar US output and unemployment rate in logs detrended with the Hodrick-Prescott filter. Clearly, unemployment oscillates more than output: The standard deviation of unemployment is nine times higher than that of output. But the canonical search and matching model, the workhorse model of the labor market, generates a standard deviation of unemployment lower than that of output (Shimer, 2005). Motivated by this *unemployment volatility puzzle*, in a novel way, we introduce unemployment into a real business cycle (RBC) model. Our model generates a standard deviation of unemployment (and output) in line with data.

We study unemployment oscillations in a model where workers differ in their skills and firms love variety of workers' skills. In this extension of an RBC model, unemployment emerges because of two reasons: (i) as the job separation rate plays a minor role in US unemployment oscillations (Hall (2005a) and Shimer (2012a)), we follow much of the search and matching literature and assume that a constant share of employed workers lose their jobs every period; and (ii) we assume that job creation is costly.

The costs of job creation are supported by the household: Each period, the household decides the number of workers that are coordinated with firms. By creating jobs, households benefit in two ways. Firstly, a new job implies that an additional worker is employed and generates labor income. Secondly, because workers differ in their skills, the household exploits monopolistic profits from each employed worker by setting a markup over the marginal rate of substitution of consumption for leisure. The household creates jobs until it balances the costs and benefits of job creation.

Our model generates highly volatile unemployment. A positive productivity shock increases wages because of labor demand pressure. In our model, because the households create jobs, higher wages increase the incentives for job creation: the household exploits a larger economic surplus of creating an additional job

after a positive productivity shock. As a result, job creation and unemployment volatility increase. This is not the case in search and matching models. In these models, the increase of wages in the aftermath of a positive productivity shock discourages firms to create jobs, thus generating lower unemployment volatility.

Our model has an additional endogenous amplification mechanism. In equilibrium, our model exhibits increasing returns to scale to specialization: intuitively, a higher employment rate enables workers to specialize in specific tasks that are specific to their skills, enhancing productivity. Thus, when in the aftermath of a positive productivity shock, employment increases, firms have further incentives to hire workers. This amplifies the effects explained above: it increases upward pressure in wages, further motivating households to create jobs.

Our model also outperforms the canonical RBC model, and solves two of its main problems: the lack of output volatility and the lack of output persistence (King and Rebelo (1999)). The high unemployment volatility generated by our model propagates to output, increasing output volatility. This is achieved even without a Frisch elasticity inconsistent with micro studies.<sup>1</sup> Our extension of an RBC model is also able to increase persistence of output: even after 25 years past a productivity shock, the shock still has visible effects on output; yet, in the canonical RBC model, the shock has a neglectable effect 20 years past its emergence.

The chapter is structured as follows. In Section 3.2, we review the literature on the unemployment volatility puzzle. We proceed with Section 3.3 by showing our modeling assumptions and with Section 3.4 by calibrating our model. In Section 3.5, we show how our model solves the unemployment volatility puzzle, and how it outperforms the canonical RBC model for both output's volatility and persistence. In Section 3.6, we conclude.

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<sup>1</sup>For more details on the difference between micro and macro labor supply elasticities, see Chetty et al. (2011).

## 3.2 The Unemployment Volatility Puzzle Literature

Shimer (2005) shows that the unemployment volatility in the canonical search and matching model is 20 times smaller than in data (18 times smaller in the search and matching model used in this chapter). The difference in results between our model and search and matching models reflect differences in the job creation process. In the canonical search and matching model, unemployed workers search for jobs every period, while firms open vacancies. Opening vacancies is costly: Firms contrast the expected value of a vacancy –which depends on the value of a filled job and the likelihood of filling a vacancy– with its costs. Furthermore, a matching function determines the number of matches. This matching function is homogeneous of degree one and concave, and determines the number of matches as a function of unemployed workers and vacancies. A job is created if, upon meeting, both worker and firm agree on a wage.

The unemployment volatility puzzle of search and matching models results from (i) the interaction of congestion externalities (concave matching function) with vacancy posting costs and from (ii) the wage setting mechanism. A positive productivity shock induces firms to open more vacancies. But, as the number of vacancies increases, it is harder to fill each vacancy and, thus, more costly to hire a worker. At the same time, because an unemployed worker can more easily find a job, wages increase. These two effects together pin down the effect of higher labor productivity on unemployment.

Many researchers, all remaining in the framework of search and matching models, have tried to solve the unemployment volatility puzzle. Hall (2005b) and Shimer (2004) advocate that real wage rigidities can help solve the unemployment volatility puzzle by removing the effect of productivity shocks on wages. Hall and Milgrom (2008) advocate an alternative wage bargain with a threat to keep bargaining next period. With this assumption, wages become less sensitive to market conditions.

Solutions advocating real wage rigidity, however, have been criticised on em-

pirical grounds. In search and matching models, job creation is determined by the wage of new hires. And Pissarides (2009), Haefke et al. (2008), and Carneiro et al. (2012) found no evidence that the wages of new hires are rigid; they concluded that the wages of new hires are as cyclical as labor productivity.

Hagedorn and Manovskii (2008), on the other hand, advocate changes to the conventional calibration of search and matching models. They advocate that the workers' bargaining power and the workers' surplus achieved by moving from unemployment to employment should be close to null. This calibration strategy implies a lower firms' surplus of hiring a new worker, increasing the sensitivity (in percentage terms) to productivity. Although this calibration strategy does increase the volatility of the canonical search and matching model, it has also been criticised on the empirical validity of their assumptions: Costain and Reiter (2008) show that by simply calibrating the canonical search and matching model, it is impossible to obtain both a reasonable response to productivity shocks and a reasonable response to policy variables. Thus, by obtaining one, Hagedorn and Manovskii (2008) neglect the other.

In this chapter, we do not try to build a search and matching model without an unemployment volatility problem. Instead, we propose an alternative method to model unemployment in an RBC model. And, in particular, we propose an alternative model that has a behavior consistent with data for two main variables: unemployment and output.

### **3.3 The Model**

Our model is a simple extension of an RBC model. There are two agents in our model, firms and households. Firms produce the final good by employing capital,  $k_t$ , and a labor composite of all available labor types,  $l_t$ . Households consume the final good, rent capital, and supply labor. The labor supply in our model differs from the canonical RBC model. In the canonical RBC model, workers are homogeneous and the household only supplies labor through the intensive

margin of labor. In our model, households supply workers with differentiated skills and through both the intensive and extensive margins of labor. Each period, the household decides hours per worker and the number of new jobs,  $x_t$ . The number of new jobs is determined by the costs of job creation and by the economic benefits of employing an additional worker. It is used as a control variable for employment,  $n_t$ , which follows the law of motion

$$n_t = (1 - \delta_n)n_{t-1} + x_t, \quad (3.1)$$

where  $\delta$  is a constant fraction of jobs that are exogenously destroyed each period.

### 3.3.1 Representative Firm

Firms produce the final good, which can be used for consumption, investment, and the costs of job creation. To produce the final good, firms combine capital and labor in the form

$$y_t = a_t k_t^\alpha l_t^{1-\alpha}, \quad (3.2)$$

where  $\alpha$  is the capital share, and  $a_t$  is a common productivity factor. The labor input,  $l_t$ , is a composite of hours per worker,

$$l_t = \left[ \int_{j \in J} h_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \quad (3.3)$$

Each worker  $j$  supplies a differentiated skill, and if employed, works  $h_t(j)$  hours. Because of unemployment, at any given time  $t$ , only a subset of all labor types,  $J$ , is available. We denote this subset by  $J_t \subset J$ . Finally, the parameter  $\theta > 0$  governs the elasticity of substitution between differentiated labor types.

The first order conditions to the firm's cost minimization problem are

$$r_t = \alpha y_t / k_t, \quad (3.4)$$

$$w_t = (1 - \alpha) y_t / l_t, \quad (3.5)$$

$$w_t = \left[ \int_{j \in J_t} w_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \quad (3.6)$$

$$h_t(j) = \left( \frac{w_t(j)}{w_t} \right)^{-\theta} l_t, \quad (3.7)$$

where  $r_t$  is the rental rate of capital;  $w_t$  is the aggregate hourly wage; and  $w_t(j)$  is the hourly wage for the labor services of the worker  $j$ . These first order conditions are similar to the ones of RBC models extended with worker's differentiated skills (e.g. Chari et al. (2007)). The only difference lies on the fact that our model features unemployment, whereas other models assume full employment.

### 3.3.2 Representative Household

Households own the firms. Each household is composed of a large number of members of total measure unity who are all willing to work.<sup>2</sup> But, because of unemployment, a fraction  $n_t$  is employed and a fraction  $u_t = 1 - n_t$  is unemployed. We follow Merz (1995) and assume that the household completely insures its members against employment risk, implying that members equally share consumption.

The household decides the number of jobs created each period,  $x_t$ . Job creation is an investment: It implies a cost at the time jobs are created,  $\phi(1 + \frac{1}{2}x_t)$ , but it also generates income in the following periods,  $w_t(j)h_t(j)$ . The only difference with respect to capital investment is that job creation has a direct effect on the disutility of labor,  $\frac{h_t(j)^{1+\psi^{-1}}}{1+\psi^{-1}}$ .

We incorporate these changes to an otherwise standard household's problem:

$$\max_{c_t, h_t, k_{t+1}, n_t, x_t, w_t(j)} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln c_t - \int_0^{n_t} \chi \frac{h_t(j)^{1+\psi^{-1}}}{1+\psi^{-1}} dj \right]$$

---

<sup>2</sup>Given our focus on the U.S. business cycle fluctuations, and the lack of cyclicalities in the U.S. labor force, this assumption does not compromise our results (Rogerson and Shimer, 2011).



subject to the budget constraint, labor demand, and the law of motion of employment

$$\int_0^{n_t} w_t(j) h_t(j) dj + (r_t + (1 - \delta_k))k_t \geq c_t + k_{t+1} + \phi \left( x_t + \frac{1}{2} x_t^2 \right),$$

$$h_t(j) \geq \left( \frac{w_t(j)}{w_t} \right)^{-\theta} l_t,$$

$$n_t \leq (1 - \delta_n) n_{t-1} + x_t,$$

where  $\beta$  is the discount factor;  $\chi$  is a measure of the disutility of labor;  $\psi$  is the Frisch elasticity of labor supply;  $\delta_k$  is the capital depreciation rate; and  $\phi$  is a scale parameter of the costs of job creation.

We anticipate symmetric equilibrium across labor types: All employed workers work the same number of hours,  $h_t$ , and earn the same wage,  $w_t^h$ . Then, the first order conditions are

$$w_t^h = \frac{\theta}{\theta - 1} \chi h_t^{\psi-1} c_t, \quad (3.8)$$

$$1 = \beta E_t \left[ \frac{c_t}{c_{t+1}} (1 - \delta_k + r_{t+1}) \right], \quad (3.9)$$

$$\phi(1 + x_t) = \frac{\theta + \psi}{\theta(\psi + 1)} h_t w_t^h + \beta(1 - \delta_n) E_t \left[ \frac{c_t}{c_{t+1}} \phi(1 + x_{t+1}) \right]. \quad (3.10)$$

The first two equations are the same as in RBC models extended with worker's differentiated skills: Households set a constant mark-up over the marginal rate of substitution of consumption for labor, and invest in capital until they are indifferent between consuming one unit this period or investing and consuming it next period. The third equation, however, is one of the novelties of this chapter. It is the *job creation equation*: The marginal cost of creating a job must equal the sum of (i) the marginal gain of an additional employed worker for the household in consumption units and (ii) the continuation value of employment.

### 3.3.3 Symmetric Equilibrium and Aggregation

In a symmetric equilibrium, Eqs. 3.3 and 3.6 imply<sup>3</sup>

$$l_t = h_t n_t^{\frac{\theta}{\theta-1}}, \quad (3.11)$$

$$w_t = w_t^h n_t^{\frac{1}{1-\theta}}. \quad (3.12)$$

These two equations are not standard in the RBC literature: The labor composite is not a linear function of employment, as well as the wage index is not equal to the hourly wage. But they are a natural result of two assumptions of our model. They result from the assumption of firms love-for-variety of skills and from the existence of unemployment. These assumptions generate increasing returns to scale in the form of labor division: As more workers are employed, the ability to distribute tasks that fit each skill is enhanced leading to higher productivity.<sup>4</sup>

Another aggregate consistency condition is the resource constraint:

$$y_t = c_t + k_{t+1} - (1 - \delta_k)k_t + \phi \left( x_t + \frac{1}{2}x_t^2 \right). \quad (3.13)$$

This equation states that final good production is devoted to consumption, investment, and costs of job creation. A competitive equilibrium for this economy is thus described by a set of allocations  $c_t, h_t, k_t, l_t, n_t, x_t, y_t$ , prices  $r_t, w_t, w_t^h$ , and common productivity factor  $a_t$  satisfying equations 3.1, 3.2, 3.4, 3.5, and 3.8-3.13, given an exogenous process for the common productivity factor

$$\log(a_t) = \rho \log(a_{t-1}) + \varepsilon_t, \quad (3.14)$$

and the initial condition for employment, capital, and productivity:  $n_{-1}, k_{-1}$  and  $a_{-1}$ .  $\varepsilon_t$  is an i.i.d disturbance with mean zero.

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<sup>3</sup>For details on how to obtain these two equations see Melitz (2003).

<sup>4</sup>Our results, however, are not only driven by increasing returns to scale.

Table 3.1: Benchmark Parameters

Discount factor:	$\beta = 0.99$
Rate of depreciation:	$\delta_k = 1.1^{1/4} - 1$
Rate of separation:	$\delta_n = 0.1$
Elasticity of substitution:	$\theta = 6$
Frisch elasticity:	$\psi = 1$
Capital share:	$\alpha = 0.36$
Weight in utility:	$\chi = 0.8896$
Cost parameters:	$\phi = 1.8997$
	$\gamma = 2$
Persistence of tech:	$\rho = 0.95$
Std of tech shock:	$\varepsilon = 0.0072$

### 3.4 Calibration

Our model is highly nonlinear. To proceed with the analysis, we log-linearize the model around the steady-state and calibrate it. We calibrate our model to the U.S. economy and summarize our calibration in Table 3.1. We set each period  $t$  as a quarter. Accordingly, we follow much of the business cycle literature and set  $\beta = 0.99$ ,  $\delta_k = 0.025$ ,  $\alpha = 0.36$ ,  $\rho = 0.979$ , and  $var(\varepsilon) = 0.0072$ . We set  $\delta_n = 0.1$  to be consistent with the large flows out of employment in U.S. data. The choice over the Frisch labor supply elasticity,  $\psi$ , is controversial. Nevertheless, as this discussion goes beyond the scope of this chapter, we set  $\psi = 1$  in our benchmark calibration, and assess the role of  $\psi$  in our sensitivity analysis. We follow the same strategy for  $\theta$  and set  $\theta = 6$  in our benchmark calibration. Finally, we use  $\phi$  and  $\chi$  to target a steady-state unemployment rate of 6% (approximately the unemployment rate in the U.S. in the postwar period) and to normalize hours worked to one.

## 3.5 Results

### 3.5.1 Second moments

In Table 3.2, we contrast the business cycle statistics of eight key macroeconomic variables estimated using U.S. data, with the ones generated by our model, by a search and matching model, and by an RBC model. The U.S. data we use starts at 1951:1 and ends at 2012:4.<sup>5</sup> To generate the results of the three models, we assume that productivity shocks are the only driver of business cycles.<sup>6</sup> To produce all business cycle statistics, we use a Hodrick-Prescott filter with smoothing parameter of 1600.

We conclude from Table 3.2 that our model delivers business cycle statistics close to the ones observed in the US economy. We also conclude that our model outperforms its two competitors in replicating US data. If we first consider unemployment, we confirm the unemployment volatility puzzle of search and matching models: In data, the standard deviation of unemployment is 13.10, whereas the search and matching model generates a standard deviation of 0.71. Our model, on the other hand, generates a standard deviation of unemployment of 12.43, very close to its data counterpart.

The unemployment volatility puzzle of search and matching models, first documented by Shimer (2005), results from (i) the interaction of congestion externalities (concave matching function) with vacancy posting costs and from (ii) the wage setting mechanism. A positive productivity shock induces firms to open more vacancies. But the decrease in the vacancy filling ratio and the increase in wages pin down the effect of higher labor productivity on unemployment. Our model, however, does not have an unemployment volatility problem. Eq. 3.10 implies that an increase in wages encourages the households to create more jobs, thus

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<sup>5</sup>GDP, consumption, and investment are real and in per capita terms. Wages refer to the non-farm real compensation per hour. Consumption comprises both nondurables spending and services. Investment comprises both investment and durables spending. The data are from the Federal Reserve Economic Data.

<sup>6</sup>The two models are detailed in the appendix as well as the calibrations we used to obtain the results reported in Table 3.2.

Table 3.2: Selected Moments

A. Standard Deviations								
	$y$	$u$	$n$	$h$	$nh$	$w(z)$	$i$	$c$
US Data (1951:1–2012:4)	1.51	13.10	0.84	0.52	1.75	0.89	6.17	0.85
Benchmark Model	1.53	12.43	0.76	0.40	0.90	0.66	2.65	0.43
Search Model	1.23	0.71	0.04	0.43	0.46	0.78	3.81	0.39
RBC Model	1.22	–	–	–	0.44	0.79	3.83	0.39
B. Relative Standard Deviations								
	$y$	$u$	$n$	$h$	$nh$	$w(z)$	$i$	$c$
US Data (1951:1–2012:4)	1.00	8.68	0.56	0.35	1.16	0.59	4.09	0.56
Benchmark Model	1.00	8.11	0.50	0.26	0.59	0.43	1.73	0.28
Search Model	1.00	0.57	0.04	0.35	0.37	0.64	3.09	0.32
RBC Model	1.00	–	–	–	0.36	0.65	3.13	0.32
C. Cross Correlations								
	$y$	$u$	$n$	$h$	$nh$	$w(z)$	$i$	$c$
US Data (1951:1–2012:4)	1.00	-0.79	0.80	0.71	0.84	0.17	0.88	0.75
Benchmark Model	1.00	-0.82	0.82	0.66	0.98	0.97	0.78	0.88
Search Model	1.00	-0.88	0.88	0.98	0.98	0.99	0.99	0.90
RBC Model	1.00	–	–	–	0.98	0.99	0.99	0.90
D. Autocorrelations								
	$y$	$u$	$n$	$h$	$nh$	$w(z)$	$i$	$c$
US Data (1951:1–2012:4)	0.81	0.89	0.90	0.81	0.89	0.73	0.84	0.78
Benchmark Model	0.77	0.93	0.93	0.70	0.84	0.65	0.94	0.82
Search Model	0.73	0.25	0.25	0.74	0.76	0.74	0.73	0.80
RBC Model	0.72	–	–	–	0.71	0.73	0.71	0.80

*Note:* This table shows the standard deviation, relative (to output) standard deviation, cross correlation with output, and autocorrelation of 8 variables of interest in data, our model, search and matching model, and RBC model. The variables of interest are output, unemployment, employment, hours per worker, total hours, wage, investment, and consumption.

increasing the sensitivity of unemployment to productivity shocks. Furthermore, firms' love for variety of skills generates increasing returns to scale in equilibrium which further contributes to higher volatility in our model.

Our extension of an RBC model contributes to solving one of the canonical RBC model's main problems: The lack of output volatility (see King and Rebelo (1999)). If we contrast the standard deviation of GDP generated by our model and generated by its competitors, we easily conclude that our model does a better job in replicating US data: The volatility of GDP in our model is 1.53, close to 1.51 in data, and far above the volatility of GDP generated by the RBC model and by the search and matching model, 1.22 and 1.23. This is the case because the high unemployment volatility in our model amplifies the effect of productivity shocks on GDP. As for the volatility of total hours (other of the canonical RBC's main problems), our model, although not completely satisfying, does a better job in generating higher volatility: Our model generates about half of the volatility in total hours we observe in data. For the remaining variables, our model does a fair job even though it generates a smaller volatility for investment, partly due to the two types of investment in our model.

The models generate similar cross correlations with output. The exception is for the case of hours per worker. In our model, the cross correlation of hours per worker and output is below its data counterpart, while in the search and matching model it is above. Regarding autocorrelations, except for wages per hour, our model generates more persistence of the effects of productivity shocks. This is particularly the case for the persistence of GDP, for which our model generates a auto-correlation of 0.77 close to 0.81 in data. Our model also generates more reasonable autocorrelations for unemployment and employment than the search and matching model. For these two variables, the search and matching model generates a staggering 0.25 autocorrelation, way below 0.88 and 0.89 (respectively) in data.<sup>7</sup>

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<sup>7</sup>This only holds with quarterly calibration. With monthly calibration, standard in the search and matching literature, the autocorrelation of unemployment and employment in the search and matching model are close to their data counterparts.

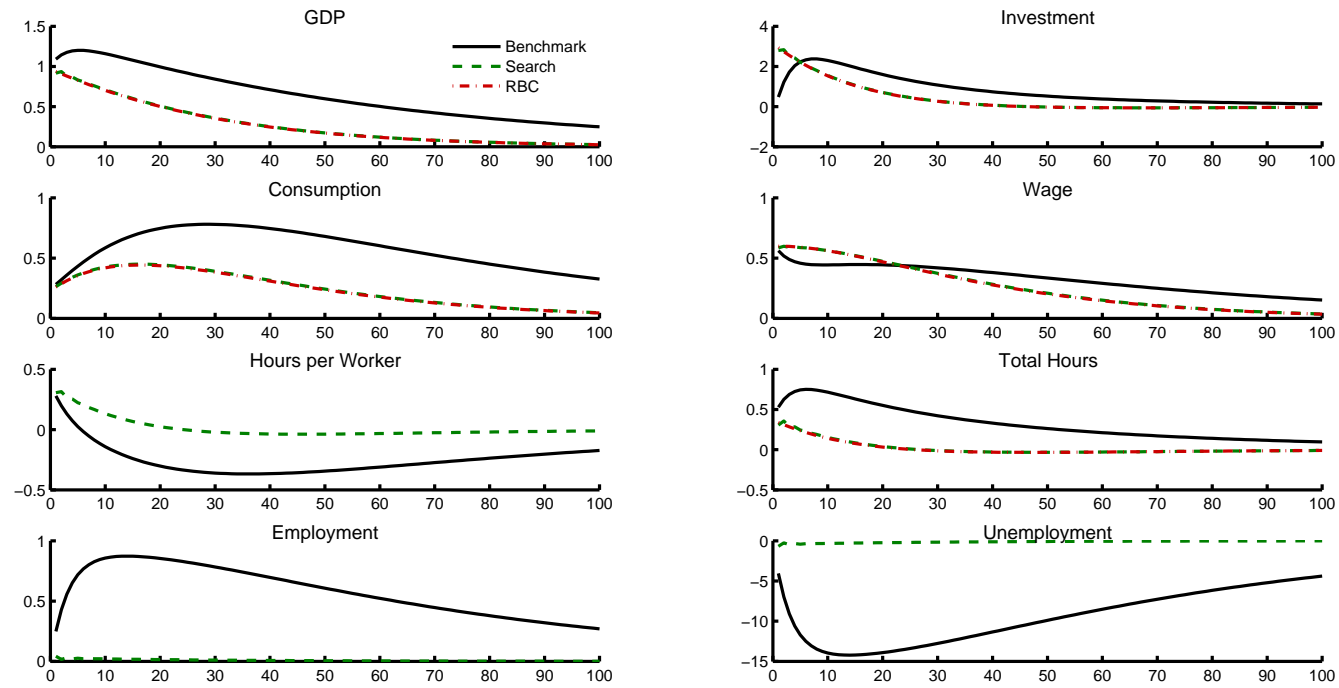
### 3.5.2 Impulse Response Functions

To gain further intuition, we proceed by contrasting the impulse response functions (IRF) of our model with those of the RBC model and the search and matching model (see Figure 3.2). The IRF confirm our main results from Table 3.2: In response to a productivity shock, (i) unemployment in our model is much more volatile than in the search and matching model, and (ii) GDP in our model has a stronger and more persistent response than in both RBC and search and matching models. One other interesting result is that the search and matching model and the RBC model plots almost overlap each other: The search and matching model is unable to amplify the effect of productivity shocks.

The response of unemployment and employment in our model is overwhelmingly higher than in the search and matching model. Although unemployment and employment do respond in the search and matching model, their response is insignificant near the response in our model: In the figure, it is almost a flat line over zero; our model, however, predicts that unemployment falls by 15% two and half years after the shock. Regarding GDP, both the impact and the persistence are higher in our model. In our model, GDP is clearly above trend even after 25 years after the productivity shock. In the RBC and search and matching models, 20 years past the productivity shock, the GDP is close to trend.

In our model, total hours increase significantly in response to the productivity shock even though hours per worker becomes negative after one year. The response of consumption in our model is also much higher than in the search and matching model and in the RBC model: In our model, consumption increases by 0.8% in its peak, whereas in the two competitors it increases by about 0.4%. The magnitude of response of wages and investment do not differ significantly across the three models.

Figure 3.2: Impulse Response Functions



*Note:* This figure plots the impulse response function of our model (full line), of the search and matching model (dashed line), and of the RBC model (dash dot line) to a 1% productivity shock.



### 3.5.3 Sensitivity Analysis

The values assigned to some of our model's parameters are not standard in the literature. Therefore, we proceed with a sensitivity analysis of the results of our model. In Table 3.3, we display the standard deviation of GDP, and the standard deviations of the remaining seven variables relative to the one of GDP. We assess the robustness of the results of our model to changes in the Frisch elasticity of labor supply,  $\psi$ , and in the elasticity of substitution between labor types,  $\theta$ .

A staggering result of our model is that the large volatility of unemployment relative to output is robust to changes in  $\psi$  and  $\theta$ : Our model does not have an unemployment volatility problem. The standard deviation of unemployment relative to the standard deviation of GDP is about 8, very close to the 8.68 in US data. The standard deviation of the remaining variables relative to the standard deviation of GDP are also quite robust to changes in  $\psi$  and  $\theta$ .

The standard deviation of output is, however, slightly sensitive to changes in  $\psi$  and  $\theta$ . In any case, it is always larger than the standard deviation of GDP in the search and matching model and in the RBC model calibrated with the benchmark calibration. This suggests that our model has a strong amplification mechanism that is robust to changes in  $\psi$  and  $\theta$ . Even with a small Frisch elasticity, 0.2 consistent with micro studies (see Chetty et al. (2011)), our model is able to generate a standard deviation 10% higher than the RBC model with benchmark calibration.

## 3.6 Concluding Remarks

In this chapter, our research agenda was to build a model that generates high unemployment volatility, as we observe in US data. For that, we extended an RBC model to feature unemployment in a novel way. In our model, workers differ in their skills, firms love variety of skills, and households incur in costs of job creation.

Table 3.3: Sensitivity Analysis

	Relative Standard Deviations							
	$y$	$u$	$n$	$h$	$nh$	$w^h$	$i$	$c$
US Data (1951:1–2012:4)	1.51	8.68	0.56	0.35	1.16	0.59	4.09	0.56
Baseline Calibration	1.53	8.11	0.50	0.26	0.59	0.43	1.73	0.28
$\psi = 0.2$	1.32	8.04	0.49	0.09	0.51	0.58	1.81	0.28
$\psi = 0.5$	1.42	8.09	0.50	0.17	0.55	0.50	1.69	0.28
$\psi = 4$	1.78	8.05	0.49	0.43	0.68	0.33	2.19	0.28
$\theta = 3$	1.67	8.44	0.52	0.22	0.57	0.45	1.74	0.33
$\theta = 25$	1.46	7.97	0.49	0.27	0.59	0.43	1.80	0.27
$\theta = 100$	1.44	7.94	0.49	0.28	0.59	0.43	1.82	0.27

*Note:* This table shows the sensitivity of the results of our model to changes in the Frisch elasticity of labor supply,  $\psi$ , and in the elasticity of substitution between labor types,  $\theta$ .

Our model is able to overcome the unemployment volatility problem of search and matching models (Shimer (2005)), generating unemployment volatility consistent with US data. Our model also outperforms the canonical RBC model by solving two of its main drawbacks: lack of output volatility and lack of output persistence (King and Rebelo (1999)).

# Appendix A

## Real Business Cycle Model

In this appendix, I explain the real business cycle model used in Chapter 3.

### A.1 Households

The household's utility maximization problem is the following:

$$\max_{c_t, h_t, k_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln c_{t+i} - \chi \frac{h_{t+i}^{1+\psi^{-1}}}{1+\psi^{-1}} \right]$$

subject to

$$d_t + w_t h_t + (r_t + (1 - \delta_k))k_t \geq c_t + k_{t+1},$$

The first order conditions are

$$w_t = \chi h_t^{\psi^{-1}} c_t \tag{A.1}$$

$$1 = \beta E_t \left[ \frac{c_t}{c_{t+1}} (1 - \delta_k + r_{t+1}) \right]. \tag{A.2}$$

## A.2 Firms

The representative firm produces the final good using capital and labor:

$$y_t = a_t k_t^\alpha h_t^{1-\alpha}. \quad (\text{A.3})$$

Cost minimization implies

$$r_t = \alpha y_t / k_t, \quad (\text{A.4})$$

$$w_t = (1 - \alpha) y_t / h_t. \quad (\text{A.5})$$

## A.3 Equilibrium

To complete the model, we add the resource constraint

$$y_t = c_t + k_{t+1} - (1 - \delta_k) k_t. \quad (\text{A.6})$$

## A.4 Calibration

We use  $\chi$  to target steady state hours:  $h = 1$ . We follow the real business cycle literature to calibrate the remaining parameters:  $\beta = 0.99$ ,  $\alpha = 0.36$ ,  $\delta_k = 1.1^{1/4} - 1$ , and  $\psi = 1$ . The parameters regarding the productivity shock are the same as for the main model of Chapter 3.

# Appendix B

## Search and Matching Model

In this appendix, I explain the search and matching model used in Chapter 3. We assume that the law of motion of employment is

$$n_{t+1} = (1 - \delta_n)n_t + m_t, \quad (\text{B.1})$$

where  $m_t$  is the number of new matches. The number of new matches is determined by a matching function which depends on two arguments, vacancies and unemployed workers:

$$m_t = \sigma(1 - n_t)^\eta v_t^{1-\eta}, \quad (\text{B.2})$$

where  $\sigma$  is a scale parameter;  $\eta$  is the elasticity of the matching function with respect to unemployment; and  $v_t$  is the number of vacancies. The job finding probability is

$$f(\theta_t) = \sigma\theta_t^{1-\eta}, \quad (\text{B.3})$$

while the job filling probability is

$$q(\theta_t) = \sigma\theta_t^{-\eta}, \quad (\text{B.4})$$

where  $\theta_t \equiv \frac{v_t}{1-n_t}$  is the labor market tightness.

## B.1 Households

Let  $V_t$  denote the household's lifetime utility. Taking as given the probability that the household's members find a job, the objective of the household is to choose a path for  $c_t$  that maximizes

$$V_t = \max_{\{c_t\}} \left[ \log c_t - \gamma n_t \frac{h_t^{1+\psi}}{1+\psi} + \beta E_t V_{t+1} \right], \quad (\text{B.5})$$

subject to

$$c_t = w_t n_t h_t + T_t + \Pi_t, \quad (\text{B.6})$$

$$n_{t+1} = (1 - \delta_n) n_t + f(\theta_t)(1 - n_t), \quad (\text{B.7})$$

where  $\Pi_t$  are profits stemming from owning the firms.

The maximization problem of the household implies that profits are discounted by

$$\Lambda_{t+1} \equiv \beta E_t \frac{c_t}{c_{t+1}}. \quad (\text{B.8})$$

## B.2 Firms

Each firm employs  $n_t$  workers, and produces output,  $y_t$ , by means of the Cobb-Douglas production function

$$y_t = a_t k_t^\alpha (n_t h_t)^{1-\alpha}. \quad (\text{B.9})$$

Firms decide the number of vacancies to open each period,  $v_t$ . For each vacancy open, firms pay a cost  $\kappa$ . Firms have the ability to transform final goods into capital goods by means of a linear technology. Capital goods depreciate at a rate  $\delta_k$  per period. Let  $J_t$  denote the value of the representative firm. The firm's

objective is to choose a path for  $v_t$  and for  $k_{t+1}$  that maximizes

$$J_t = \max_{v_t, k_{t+1}} \left( a_t k_t^\alpha (n_t h_t)^{1-\alpha} + (1 - \delta_k) k_t - k_{t+1} - w_t n_t h_t - v_t \kappa + E_t [\Lambda_{t+1} J_{t+1}] \right), \quad (\text{B.10})$$

subject to

$$n_{t+1} = (1 - \delta_n) n_t + q(\theta_t) v_t. \quad (\text{B.11})$$

Let  $J_n$  denote the marginal value of a worker to the firm. Then, the first-order condition of the firm's maximization problem with respect to  $v_t$  can be expressed as

$$E_t [\Lambda_{t+1} J_{n,t+1}] q(\theta_t) = \kappa, \quad (\text{B.12})$$

while the envelop condition for employment can be expressed as

$$J_{n,t} = (1 - \alpha) \frac{y_t}{n_t} - w_t h_t + (1 - \delta_n) E_t [\Lambda_{t+1} J_{n,t+1}]. \quad (\text{B.13})$$

Let  $J_{k,t}$  denote the marginal value of capital to the firm at time  $t$ . Then, the first-order condition of the firm's maximization problem with respect to  $k_{t+1}$  can be expressed as

$$1 = E_t [\Lambda_{t+1} J_{k,t+1}], \quad (\text{B.14})$$

while the envelop condition for capital can be expressed as

$$J_{k,t} = \alpha \frac{y_t}{k_t} + 1 - \delta_k. \quad (\text{B.15})$$

We combine the last two equations by evaluating Eq. B.15 at  $t + 1$  and substituting it into Eq. B.14:

$$1 = E_t \left[ \Lambda_{t+1} \left( \alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta_k \right) \right]. \quad (\text{B.16})$$

### B.3 Wages and Hours per worker

We assume that wages and hours per worker are determined at the start of each period, and that they are the result of Nash bargaining between the worker and the

firm:

$$w_t = \arg \max_{w,h} \tilde{V}_{n,t}(w, h)^\phi \tilde{J}_{n,t}(w, h)^{1-\phi}, \quad (\text{B.17})$$

where  $\tilde{V}_{n,t}(w, h)$  is the marginal value to the household of having a worker employed at some wage  $w$  per  $h$  hours worked rather than unemployed;  $\tilde{J}_{n,t}(w, h)$  is the marginal value to the firm of employing an additional worker at some wage  $w$  per  $h$  hours worked; and  $\phi$  is the worker's bargaining power.

Following the steps in Shimer (2012a), we find that  $w_t$  and  $h_t$  satisfy the equations:

$$\begin{aligned} \phi J_{n,t} &= w_t h_t (1 - \phi) - \gamma \frac{h_t^{1+\psi}}{1 + \psi} c_t (1 - \phi) + \\ &\quad \phi (1 - \delta_n - f(\theta_t)) E_t [\Lambda_{t+1} J_{n,t+1}], \end{aligned} \quad (\text{B.18})$$

$$\gamma h_t^{1+\psi} = (1 - \alpha) \frac{y_t}{n_t c_t}. \quad (\text{B.19})$$

## B.4 Equilibrium

To complete the model, we add the resource constraint

$$y_t = c_t + k_{t+1} - (1 - \delta_k) k_t + v_t \kappa. \quad (\text{B.20})$$

## B.5 Calibration

To calibrate the model, we use  $\kappa$ ,  $\sigma$ , and  $\gamma$  to target steady-state employment, hours per workers, and job filling probability. In particular, we target  $n = 0.94$ ,  $h = 1$ , and  $q = 0.7$ . We calibrate the remaining parameters following the real business cycle literature and search and matching literature:  $\beta = 0.99$ ,  $\alpha = 0.36$ ,  $\delta_k = 1.1^{1/4} - 1$ ,  $\psi = 1$ ,  $\eta = 0.5$ ,  $\delta_n = 0.1$ , and  $\phi = 0.5$ . The parameters regarding the productivity shock are the same as for the main model of Chapter 3.



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